

# How to Design

# Analog Filter Circuits

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*Because almost every analog circuit contains some filters, understanding how to work with them is important. Here we'll discuss the basics of both active and passive types.*

THE MAIN PURPOSE OF AN ANALOG FILTER circuit is to either pass or reject signals based on their frequency. There are many types of frequency-selective filter circuits; their action can usually be determined from their names. For example, a band-rejection filter will pass all frequencies except those in a specific band. Consider what happens if a parallel resonant circuit is connected in series with a signal source. It lets all frequencies pass freely, with the exception of those in a small band of frequencies around the circuit's *resonant frequency*. (At its resonant frequency, the parallel circuit theoretically acts as an infinite impedance. That impedance is greatly reduced at off-resonant frequencies.)

On the other hand, should the parallel L-C circuit be placed across (in parallel with) the signal source, only the narrow band previously rejected would be allowed to pass from the source to the amplifier. Thus, we have just described a *bandpass* filter.

In addition to bandpass and band-rejection filters, circuits can be designed to only pass frequencies that are either above or below a certain *cutoff* frequency. If the circuit passes only frequencies that are below the cutoff, the circuit is called a *lowpass* filter, while a circuit that passes those frequencies above the cutoff is a *highpass* filter.

All of the different filters fall into one of two categories: active or passive. *Passive* filters are composed of passive components such as resistors, capacitors, and inductors. Inductors, however, are undesirable at some frequencies because of their size and/or practical performance. (At low frequencies, a real inductor's properties vary considerably from those of an ideal inductor.) *Active* filters consist of passive elements along with an active element such as a transistor or an integrated-circuit op-amp. Active filters are usually designed without using inductors. Therefore they have the advantage of being less expensive on the aver-

age (because inductors can be expensive and hard to find); they are generally easier to tune; they can provide gain (and thus they do not necessarily have any insertion loss); they have a high input impedance, and have a low output impedance.

A filter can be in a circuit with active devices and still not be an active filter. For example, if a resonant circuit is connected in series with two active devices (such as transistors) it is called a passive filter. But, if the same resonant circuit is part of a feedback loop, it is then called an active filter. We saw an example of such an active filter when we discussed a tape-player preamplifier during our discussion of feedback (see the July 1983 issue of *Radio-Electronics*).

## Basic filters

A rudimentary passive filter consists of an R-C network or an R-L network. Those networks can be found in many different types of electronic equipment performing different roles. For example,

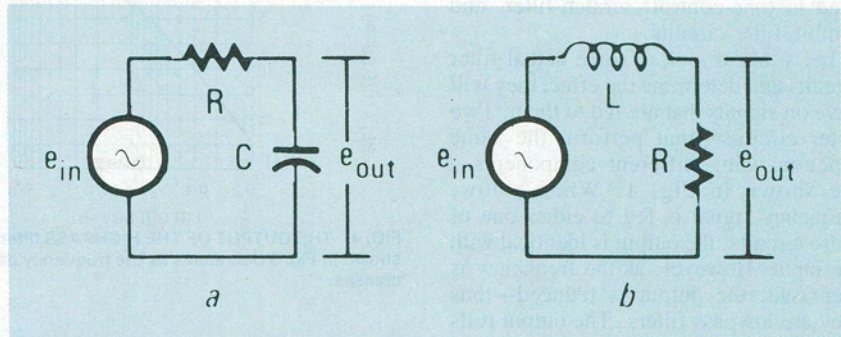


FIG. 1—TWO LOWPASS FILTERS. Even though the filters use different components, they perform in a similar fashion.

in power supplies, such circuits are used to filter undesirable ripple. In audio amplifiers, different R-C networks are used in tone-control, scratch-filter, and rumble-filter circuits.

Let's take a look at some actual filter circuits and determine the effect they will have on signals that are fed to them. Two filter circuits—that perform the same function using different components—are shown in Fig. 1. When a low-frequency signal is fed to either one of those circuits, the output is identical with the input. However, as the frequency is increased, the output is reduced—thus they are lowpass filters. The output rolls off at the rate shown by the curve in Fig. 2. The important frequency to note is  $f_o$ .

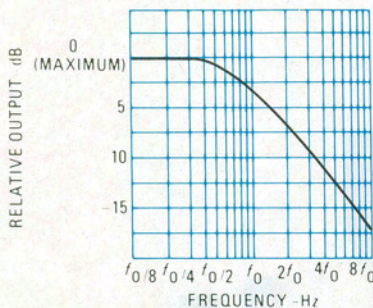


FIG. 2—THE OUTPUT OF THE LOWPASS filters shown in Fig. 1 decreases as the frequency increases.

That's the frequency where the output has dropped 3 dB from its maximum value (0.707 of the maximum). That *corner frequency* (also called the half-power point) can be determined from the values of the components in the circuit. For the R-C circuit in Fig. 1-a:

$$f_o = 1/2\pi RC. \quad (1)$$

For the R-L circuit in Fig. 1B:

$$f_o = R/2\pi L. \quad (2)$$

Figure 3 shows two circuits, either of which can be used to attenuate the *low* frequencies (thereby creating a highpass filter). Note that the curve shown in Fig. 4, when compared to Fig. 2, is symmetric about the y-axis. In Fig. 2, the output drops in the linear portion of the curve by 6-dB every time the frequency is doubled. In Fig. 4, the output drops by 6-dB every time the frequency is halved. The important point is that in the linear portion of

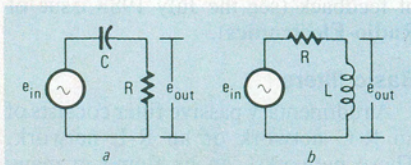


FIG. 3—TWO HIGHPASS FILTERS that use different components but perform similarly.

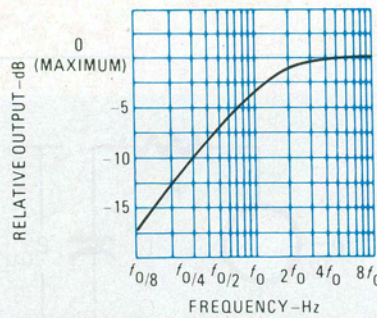


FIG. 4—THE OUTPUT OF THE HIGHPASS filters shown in Fig. 3 decreases as the frequency decreases.

each curve, the slope is the same—6-dB-per-octave.

Any of these filter circuits can be placed between two amplifier stages. If that's done, the corner frequency,  $f_o$ , is still as predicted by equation 1—presuming that the impedance of the circuit feeding the filter is zero and the input impedance of the circuit at the output of the filter is infinite. Otherwise, resistors in series with the input to the filter and in parallel with the output from the filter, must be considered when determining  $f_o$ .

For example, assume that the circuit in Fig. 3-a is connected between two transistors, as shown in Fig. 5-a. Figure 5-b is the equivalent circuit involving the filter and transistors. (The function of coupling capacitor C1 is to DC-isolate the transistor from the filter. That capacitor is usually of such magnitude that it can be treated as a short for all signal voltages.) It can be simplified by determining the equivalent resistance,  $R_o$ , of all resistances at the output. The final circuit is shown in Fig. 5-c. Equation 1 applies there also.

The analysis of this circuit—as well as more complex filter circuits that have more than one corner frequency—can be accomplished in two simple steps. First, short the input,  $e_{in}$ , and note the total resistance across the capacitor. In this

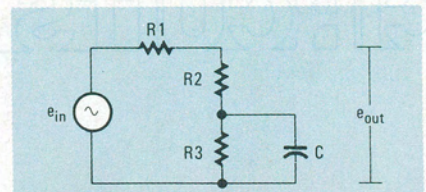


FIG. 6—A MORE COMPLEX filter circuit. This filter has more than one corner frequency.

case it is  $2.7K + R_o$ . Using equation 1, we determine that  $f_o = 1/2\pi(2.7K + R_o)C$ . The frequency at which the rolloff starts is  $f_o$ , as shown in Fig. 4.

The next step is to place a short across the output and leave  $e_{in}$  open. There is no parallel R-C combination in the circuit when that is done; the only frequency indicated in the response calculations of the filter is  $f_o$  and its curve in Fig. 4.

Now let's look at a more complex filter circuit—one in which there will be more than one corner frequency. We will use the procedure noted above to analyze the circuit in Fig. 6 and find those corner frequencies. One  $f_o$  corner frequency is the beginning of the rolloff characteristic curve and a second  $f_o$  corner frequency is on a curve with rising characteristics. The two resulting curves are then added to provide a curve illustrating the overall response of the circuit as shown in Fig. 7. First, we short  $e_{in}$  and note the total resistance across C— $R3$  in parallel with the sum of  $R1$  and  $R2$ . Substituting this information into equation 1, we find that one of the corner frequencies, where the high-frequency roll off starts, is:

$$f_{o1} = 1/2\pi ((R1 + R2) \parallel R3)C.$$

We continue by leaving  $e_{in}$  open and shorting  $e_{out}$ . Then,  $R2$  is across both  $R3$  and C. By inserting those values into equation 1 we find for the second corner frequency:

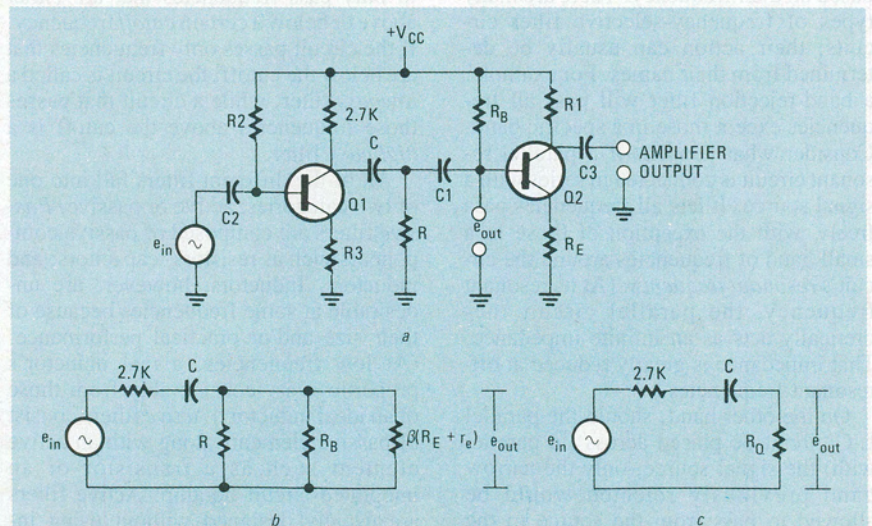


FIG. 5—TO WORK WITH a filter between two transistor circuits, you must first find the simplified equivalent circuit.

$$f_{o2} = 1/2\pi(R2\parallel R3)C$$

The curves add as shown in Fig. 7. The total curve indicates the overall response of the filter. The two equations differ only by the resistance factors. Because  $R2\parallel R3$  is smaller than  $(R1 + R2)\parallel R3$ ,  $f_{o2}$  is higher in frequency than is  $f_{o1}$ . A similar procedure may be used to determine the frequency response of other complex arrangements.

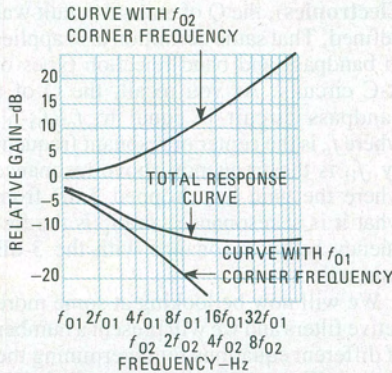


FIG. 7—FREQUENCY RESPONSE curve of the circuit shown in Fig. 6.

### Sharpening the rolloff

Rolloff using but one R-C or one R-L network, cannot exceed a 6 dB-per-octave rolloff rate. However, a faster rolloff rate can be achieved by using more than one filter in a circuit. An arrangement using two lowpass filters connected in series is in Fig. 8. There, R1-C1 is one network with a corner frequency at  $f_{o1}$ .

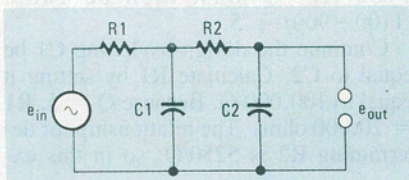


FIG. 8—TO INCREASE THE ROLLOFF RATE, you can use more than one filter in a circuit.

The combination of R2 and C2 is a second filter network, with a corner frequency at  $f_{o2}$ . Those corner frequencies are determined through use of equation 1 for each R-C combination. Curves with those corner frequencies are shown in Fig. 9. Each curve illustrates a rolloff at the rate of 6 dB-per-octave. Adding the two curves results in a characteristic with an eventual rolloff of 12 dB-per-octave.

If more resistors and capacitors are added to a circuit of the type shown in Fig. 8, a procedure similar to that used with the circuit in Fig. 6 may also be used. But this time, the circuit must be split into two portions with each containing the individual filter sections, and each section is analyzed as if the other section does not exist. The effects on the frequency response of each of

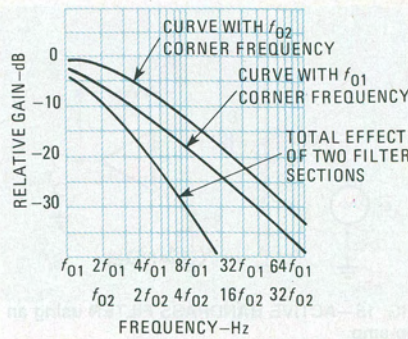


FIG. 9—THIS FREQUENCY-RESPONSE CURVE shows how the rolloff is sharpened by using more than one filter.

the two sections are then added to determine the circuit's overall response.

Other two-section filters (two R-C networks) can be combined to generate a bandpass filter circuit. That is illustrated in Fig. 10. There is low frequency rolloff due to the C1-R1 section of the filter and high frequency rolloff due to the R2-C2 filter. The corner

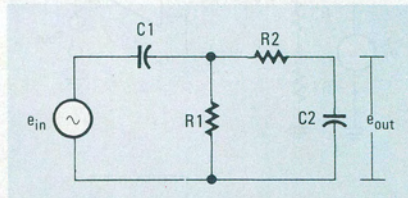


FIG. 10—BANDPASS FILTER USING two R-C networks.

frequencies,  $f_{o1}$  and  $f_{o2}$ , are determined for the R1-C1 combination and the R2-C2 combination, respectively. When  $f_{o1}$  is substantially less than  $f_{o2}$ , the response curve for the circuit is as shown in Fig. 11-a. Should  $f_{o1}$  be higher in frequency than  $f_{o2}$ , the curve in Fig. 11-b applies.

In the circuits drawn in Figs. 8 and 10, the R2-C2 combinations probably have some effect on  $f_{o2}$ . (The responses of

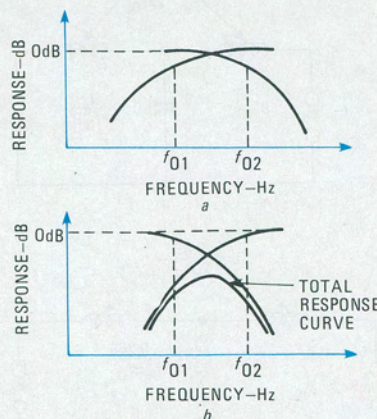


FIG. 11—FREQUENCY-RESPONSE CURVES. The response when  $f_{o1}$  is less than  $f_{o2}$  is shown in a. It is equal to that of the two original curves below the 0-dB level. Shown in b is the response when  $f_{o2}$  is less than  $f_{o1}$ .

those filters are shown in Figs. 9 and 11.) To minimize their interrelated effects, components should be chosen so that the R2-C2 circuit has a minor shunting effect on R1 and C1 and that the R1-C1 combination has only a minor affect on loading the R2-C2 combination. It is best if the two circuits were completely isolated from each other by placing each R-C filter between different transistors in a three-transistor amplifier circuit.

### Active filters

An active filter involving bipolar transistors, is shown in Fig. 12. Here we assume that  $R_E$  is much greater than  $r_e$ , the internal emitter resistance of Q2. Capacitor C1, along with the output impedance of Q1 and the input impedance of Q2, form one high pass filter. The corner frequency,  $f_{o1}$ , due to those components is

$$f_{o1} = 1/2\pi C1(R_c + \beta R_E \parallel R_B)$$

if we assume that the impedance of C2 is much higher than  $R_F$  at  $f_o$  Hz. Here,  $R_C$  is the impedance at the input of the filter,  $\beta R_E$  is the impedance reflected from the emitter circuit of Q2 into its base circuit, and  $\beta R_E \parallel R_B$ , the equivalent resistance of  $R_B$  in parallel with  $\beta R_E$ , is the resistance at the output of the filter. Due to those resistances, the output rolls off at frequencies below  $f_{o1}$ .

A second corner frequency,  $f_{o2}$ , is caused by the presence of C2 and  $R_F$  in the emitter circuit. Because C2 shunts a portion ( $R_F$ ) of the total resistance in the emitter circuit, feedback due to the presence of this resistor is reduced as the frequency increases. Hence beginning at  $f_{o2}$ , the output increases with the applied frequency. It is equal to  $1/2\pi C2 R_F$ .

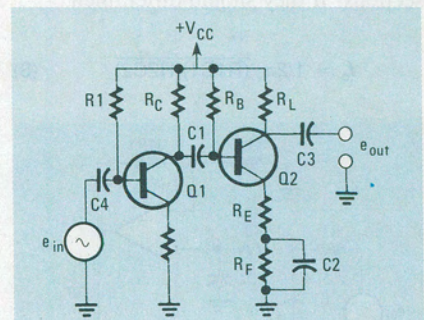


FIG. 12—ACTIVE TRANSISTOR FILTER has three corner frequencies.

Next consider the total resistance in the emitter circuit not shunted by C2. It is  $R_T = R_E + R_B/\beta$  where  $R_B/\beta$  is the resistance reflected from the base circuit into the emitter circuit. A corner frequency due to the action of those resistances along with the C2- $R_F$  combination is  $f_{o3} = 1/2\pi C2(R_F R_T / (R_F + R_T))$ . Voltage begins to roll off at that frequency.

It is obvious that  $f_{o3}$  is at a higher frequency than is  $f_{o2}$  because the resistance

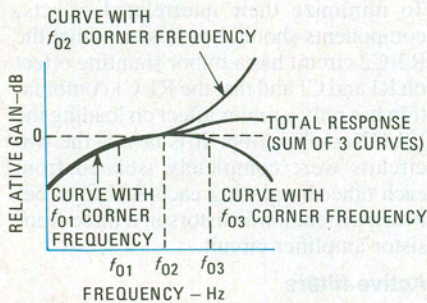


FIG. 13—CURVE SHOWING THE THREE corner frequencies and how they add to provide the overall circuit response.

in the denominator of the equation for  $f_{03}$  is lower than that for  $f_{02}$ . Usually,  $f_{01}$  is lower than either  $f_{02}$  or  $f_{03}$  because the capacitance of  $C1$  and the input and output resistors in the circuit, are all large. The value of  $R_F$  is seldom negligible when considering the factors in equation for  $f_{01}$  because  $C2$  can frequently be considered as an open circuit at  $f_{01}$  Hz.

Four curves are shown in Fig. 13. Three of these represent the response of individual filter sections in the circuit, with rolloff or amplitude increases starting at the three corner frequencies. The total curve shows the overall effect on the response of the combination of the three.

### Using op-amps

Now we'll look at an active filter that is designed around an op-amp. In the circuit shown in Fig. 14-a, the capacitor in the feedback loop attenuates the high frequencies, therefore, the circuit behaves as a lowpass filter. The frequency at which the output has been attenuated 3 dB can be approximated from equation 1 when  $R$  and  $C$  are identical in both filter sections. If they should differ, then

$$f_0 = 1/2\pi\sqrt{(R1C1)(R2C2)} \quad (3)$$

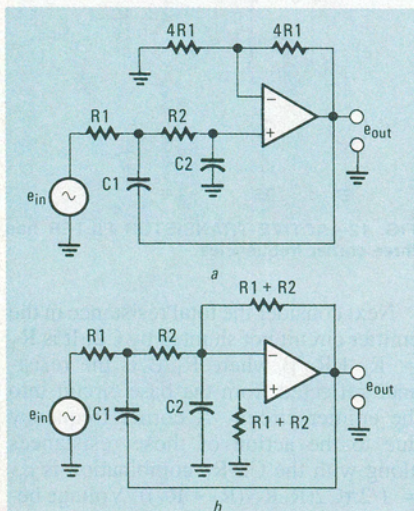


FIG. 14—ACTIVE LOWPASS FILTER. The signal is fed to the non-inverting op-amp input in a, and to the inverting input in b.

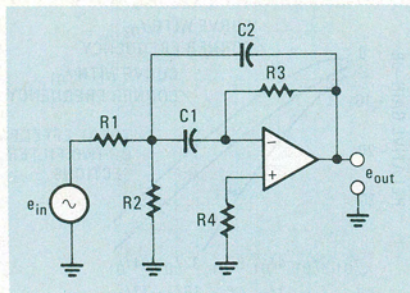


FIG. 15—ACTIVE BANDPASS FILTER using an op-amp.

where  $R1$  and  $C1$  are the components in one of the R-C networks and  $R2$  and  $C2$  are the components in the second R-C network. An inverting circuit could just as well have been used here. Figure 14-b shows such an arrangement. Equation 3 still applies. Negative feedback is applied

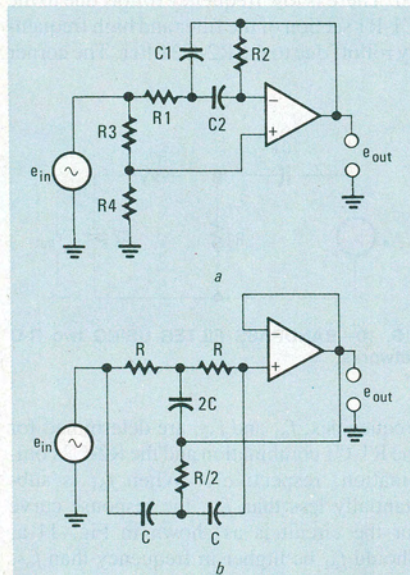


FIG. 16—ACTIVE BAND-REJECTION filters using op-amps. The filter shown in b uses a twin-T circuit.

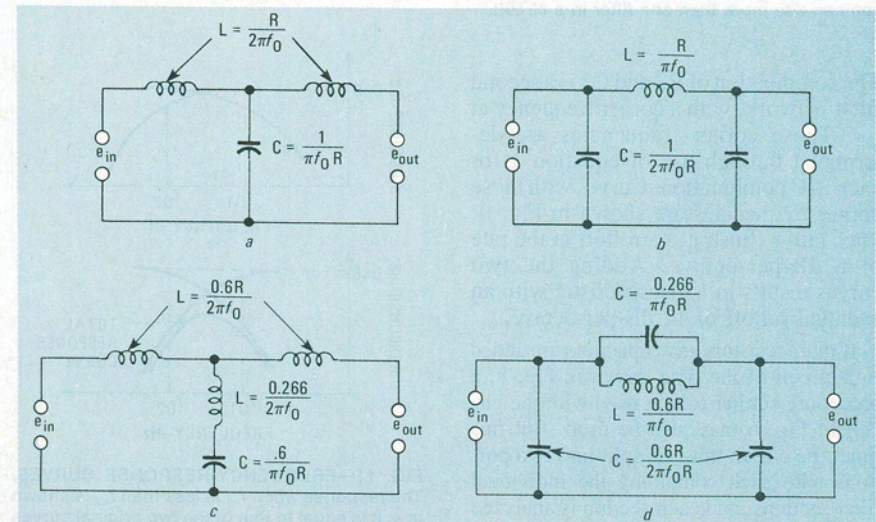


FIG. 17—LOWPASS FILTERS. The circuit shown in a is a constant-K T-section filter; b shows a constant-K  $\pi$ -section; c shows an M-derived T-section, and d shows an M-derived  $\pi$ -section filter.

through  $C1$ —high frequencies are fed through  $C1$  back to the input, to increase the rolloff rate. In both lowpass-filter circuits,  $R1$  and  $R2$  are usually made equal—about 10,000 ohms.

Highpass filters can be formed using circuits identical to those drawn in Fig. 14. All that's necessary is to interchange the locations of components in the  $R1$ - $C1$  filter as well as components in the  $R2$ - $C2$  filter. As before, equation 3 applies.

In the article on high frequency circuits (see the August 1983 issue of **Radio-Electronics**), the  $Q$  of an L-C circuit was defined. That same definition also applies to bandpass and band-rejection types of R-C circuits. As you recall, the  $Q$  of a bandpass circuit is equal to  $f_0/(f_H-f_L)$  where  $f_0$  is the center or resonant frequency,  $f_H$  is the frequency above resonance where the gain has dropped 3-dB from what it is at resonance, and  $f_L$  is the frequency below resonance with the 3-dB gain reduction.

We will now be looking at some more active filters and we will present a number of different equations for determining the various C's and R's that are needed. We will not derive the equations; the filters will be presented in "cookbook" form. You'll achieve good performance if the given relationships are observed. Rolloff will approach the ideals of 40 dB/octave for the circuits we'll discuss.

A typical bandpass filter circuit using resistors and capacitors is shown in Fig. 15. Before you determine what components you use, you must calculate just what  $Q$  must be. Let us say you want a peak response at 1000 Hz and that the gain should be down 3 dB at 900 Hz and 1100 Hz.  $Q$  must then be  $1000/(1100-900) = 5$ .

Continue the design by letting  $C1$  be equal to  $C2$ . Calculate  $R1$  by setting it equal to  $100,000/Q$ . Because  $Q = 5$ ,  $R1 = 20,000$  ohms. The relationship for determining  $R2$  is  $5250/Q$ , so in this ex-

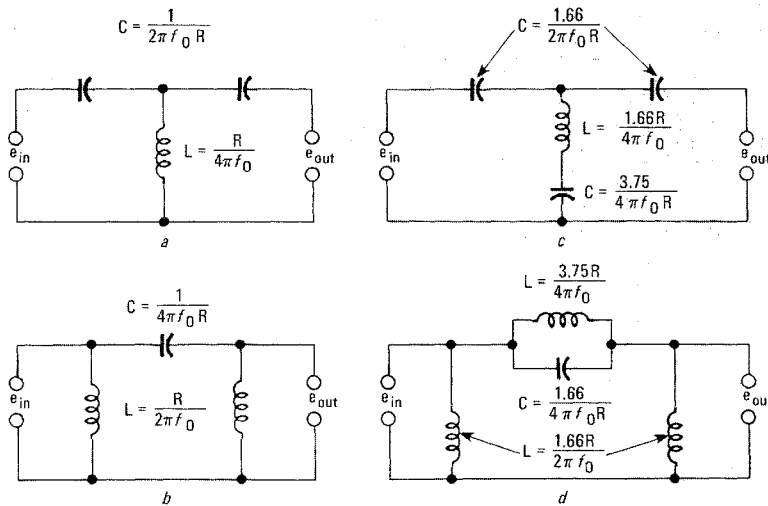


FIG. 18—HIGHPASS FILTERS. The circuit shown in *a* is a constant-K T-section filter; *b* shows a constant-K  $\pi$ -section; *c* shows an M-derived T-section, and *d* shows an M-derived  $\pi$ -section filter.

ample,  $R_2$  should be set equal to 1050 ohms. The value of  $R_3$  is equal to the product of 20,000 ohms and  $Q$ . For this problem,  $R_3 = 100,000$  ohms.

For the circuit in Fig. 15

$$f_0 = 1/2\pi \sqrt{C_1 C_2 R_3 (R_1 || R_2)} \quad (4)$$

Because  $C_1$  is identical to  $C_2$ , the product of the two capacitances is  $C^2$ . To determine  $C$ , equation 4 becomes

$$C = 1/2\pi f_0 \sqrt{R_3 (R_1 || R_2)} \quad (5)$$

Substituting  $f_0 = 1000$ ,  $R_3 = 1000,000$ , and  $R_1 || R_2 = 998$  into equation 5 yields the fact that  $C = 0.016 \mu\text{f}$ . If you follow the results, you can obtain a capable filtering circuit.

A band rejection or notch filter is in Fig. 16-a. Here,  $Q$  is determined as before. Capacitor  $C_1$  is usually made equal to  $C_2$  and the resonant frequency, as in-

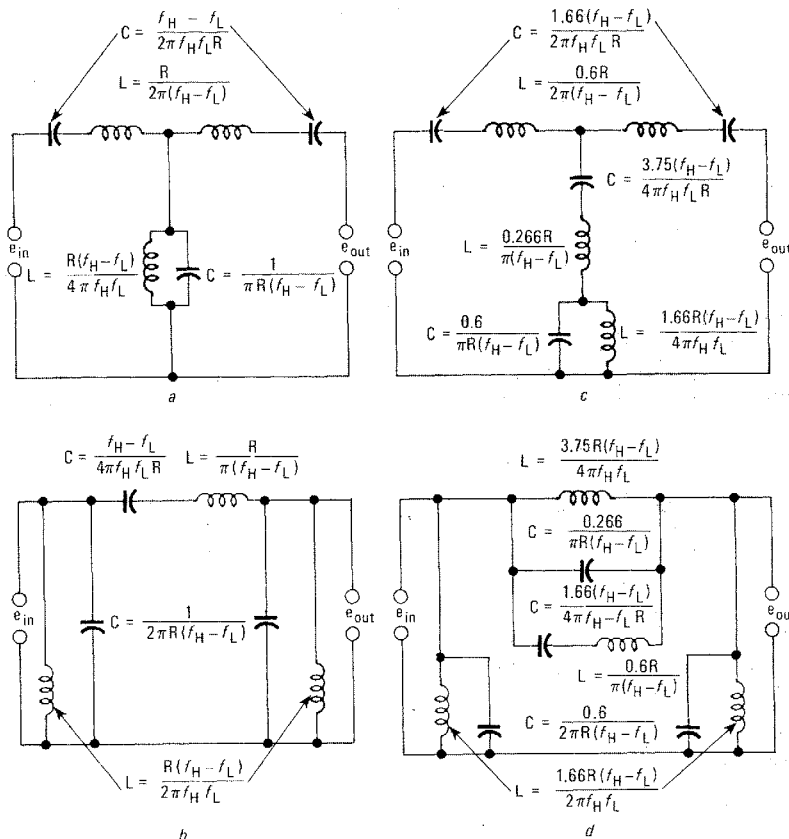


FIG. 19—BANDPASS FILTERS. The circuit shown in *a* is a constant-K T-section filter; *b* shows a constant-K  $\pi$ -section; *c* shows an M-derived T-section, and *d* shows an M-derived  $\pi$ -section filter.

indicated in equation 3, is:

$$f_0 = 1/2\pi \sqrt{R_1 R_2 C_1 C_2}$$

Once  $Q$  has been determined, the ratio of  $R_2$  to  $R_1$  in the circuit can be set equal to  $4Q^2$ . Make  $R_1$  equal to about  $100,000/Q$ ,  $R_3$  equal to about  $5250/Q$  and  $R_4$  equal to about  $50 \times R_3$ . Using this information, calculate the  $C$ 's for the circuit through use of equation 3.

Another type of notch filter (often called the twin-T arrangement) built around an op-amp is shown in Fig. 16-b. In that circuit,  $f_0$  can be determined using equation 1. Because  $Q$  can be very high, the rolloff is sharp.

### Constant-k and m-derived filters

Constant-k filters get their name from the fact that the product of the capacitive and inductance reactances ( $X_C \times X_L$ ) is constant at all frequencies. They exhibit reasonably sharp rolloff with a smooth passband. However, sharpness of rolloff can be improved considerably by using an m-derived filter.

An m-derived filter can be recognized by the parallel-resonant or series-resonant circuit in series or across the line respectively. They produce essentially infinite attenuation of the frequency to which they are tuned (thus zero transmission of that frequency along the line).

Schematics of both constant-k and m-derived filters are shown in Figs. 17 through 19.

Component values in the circuits depend on the resistance at the input to the filter circuit as well as the resistance at its output. In this discussion, we assume them to be equal—that is the usual case. We denote that resistance by the letter  $R$ . If input and output resistances differ, let  $R$  equal the average value of the resistance at the input and output. Rolloff will be affected by that mismatch. But resistance can be adjusted to the ideal by merely adding the proper transistor circuits at the input and output of the filter.

In the bandpass circuits, the  $Q$  of the coils do affect the sharpness of rolloff, but only to a minor degree. Rolloff at the low end of the pass band starts at  $f_L$  while rolloff at the high end of the passband starts at  $f_H$ . ( $f_L$  and  $f_H$  were defined above). For these circuits the resonant frequency is equal to  $\sqrt{f_H f_L}$ .

The various filter circuits shown include equations to indicate the approximate values of the components to be used in the circuits. Using those, you should be able to design a practical circuit.

Several constant-k or m-derived filter sections may be combined to sharpen rolloff and increase attenuation at different frequencies, if required. Input and output impedance matching will not be disturbed by such combinations.

Next time we'll look at another aspect of solid-state devices—how they can be used in switching applications. R-E