# Operational Amplifier Principles

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The most important function of early, high quality, dc amplifiers was to perform mathematical *operations* such as multiplication, addition, integration, and differentiation. These amplifiers became known appropriately as *operational amplifiers*, and they were used principally in analog computers. Now, with modern solid-state devices and improved assembly and packaging technique, amplifiers of this description are more economical, more reliable, and more compact than their vacuum-tube predecessors. As a result, they fill a wide diversity of needs in signal generation and signal conditioning, active filtering, measurement, and control as well as in the traditional computing functions. The list of practical applications continues to grow as the amplifiers make use of new semiconductor devices such as matched differential pairs and improved junction- and MOS field-effect transistors.

Since more and more engineers are facing problems that can be solved best by operational amplifiers, we have prepared this note as a brief refresher on the fundamental principles involved. Most of the basic material can be found in textbooks, and several good articles on the state-of-the-art have appeared in various journals during the past year or two.

## 1. INTRODUCTION

The operational amplifier is a stable, high gain, dc-coupled amplifier which is usually used with a large amount of negative feedback. In this manner the *functional amplifier circuit* is made relatively insensitive to circuit loading and the effects of temperature and time on amplifier parameters. To a good approximation, the characteristics of the amplifier in a given circuit are the characteristics of the *external feedback elements* alone, over which the designer can exercise the degree of control warranted by the application. Furthermore, by the choice of feedback elements, the designer can use a given amplifier type for dozens of different functional circuits.

To illustrate the versatility of the operational amplifier, consider the generalized functional circuit in Figure 1. The triangle symbolizes the phase-inverting amplifier *per se* of high gain A, and $Z_1$, $Z_f$ are the external feedback elements.

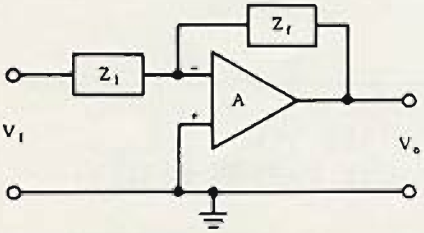


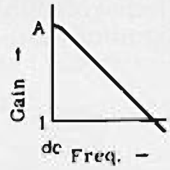

Figure 1 Generalized Circuit

- If $Z_1$, $Z_f$ are resistors $R_1$, $R_f$ the properly applied amplifier will yield an output voltage $V_o$ equal to the input signal $V_i$ *multiplied* by the ratio $R_f/R_1$.
- If $Z_1$ is a resistor and $Z_f$ is a capacitor, the circuit will *integrate* the input voltage in accordance with the externally established RC time constant.
- If $Z_f$ is a transistor, the circuit will yield the *logarithm* of the input.

In addition to these linear and non-linear operating modes, the operational amplifier is useful in the *switching* mode as a limit detector or comparator, and for circuit isolation and impedance matching.

These amplifiers will respond to *ac signals*, but the output capacitance of the transistors causes the amplifier gain to fall off as the frequency is increased. This effect is usually compensated in the multistage amplifier to produce a smooth roll-off of 6 db per octave, which renders the amplifier stable from oscillation under any value of *closed-loop* gain. (Fig. 2)

Figure 2
Open-Loop Response




The *ideal operational amplifier* would exhibit infinite input impedance so it would not load any source, and zero output impedance so it could drive any load. It would have infinite gain and bandwidth, and the output would be determined exactly by the input signal and the properties of the feedback circuit. The degree to which practical amplifiers approach the ideal is largely a matter of cost.

The principal errors that distinguish the practical operational amplifier from the ideal are the inherent *dc offset* voltage and current, which produce non-zero output with zero signal input, the *drift* of these offsets with temperature and time, and the higher frequency disturbance of similar nature which we call *noise*. Offsets, drift, and noise are *basic measurement errors*, and collectively they limit the signal resolution of the amplifier. A number of schemes have been developed to reduce their effect.

### Theme and Variations

Today's operational amplifier market comprises amplifiers identified as *differential*, *chopper-stabilized*, *FET-input*, and *linear integrated circuit*, to name those in most popular usage.* At the heart of all of these variations is a basic dc amplifier with a balanced input stage (Fig. 3) for first order temperature compensation.

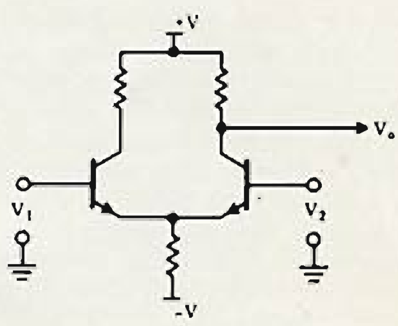



Figure 3 Differential Input Stage

* Editor's Note: Author omits varactor bridge op amps, see page 6.



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The *differential amplifier* is essentially this balanced input followed by additional stages, usually single-ended, for gain and impedance matching. Its output is a function of the difference in two inputs, hence it is particularly useful in amplifying signals from remote or otherwise isolated sources. In addition, it can be connected for non-inverting gain, which results in extremely high input impedance.

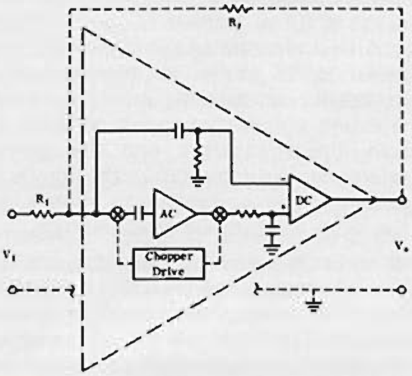


Figure 4 Chopper Stabilized Amplifier

The *chopper-stabilized amplifier* utilizes an ac-coupled "preamp" in conjunction with the external feedback loop to virtually null out the dc offsets and drift of the "main" dc amplifier. Referring to Figure 4, dc and low-frequency signals are modulated, amplified by an ac-coupled carrier amplifier, then demodulated, filtered, and fed to the differential-input of the main amplifier. Higher frequency signals are capacitively coupled directly to the dc amplifier. By careful proportioning of the circuits, the response of the preamp is superimposed on the response of the dc amplifier. The resulting over-all gain is the product of the dc gain of the "chopper channel" and the gain of the dc amplifier, over the full bandwidth of the dc amplifier. (Fig. 5)

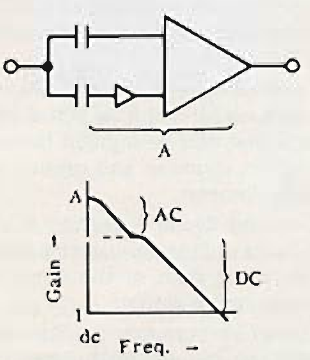


Figure 5 Chopper Amplifier Response

This technique is the most successful in reducing dc offsets and drift, and amplifiers are available with initial offsets less than ±20 microvolts and ±20 picoamperes and drift less than 0.5 μV/°C and 1 μV/week. Other types have approached this order of thermal drift *or* offset current, but where a single-ended amplifier can be used, this combination of specifications is most economically achieved by chopper-stabilization. Furthermore, the chopper-stabilized amplifier is unsurpassed for *long-term stability*.

Present-day chopper-stabilized amplifiers use only photo-resistor or transistor modulators (choppers), with the chopper drive generated internally by a multivibrator. The early mechanical chopper is still used, but short life, costly replacement, and the ac excitation voltage required spelled its demise as suitable electronic switches became available.

The *FET-input amplifier* is similar to the differential amplifier described above except for the use of field-effect transistors in the input stage. The features of this type amplifier are the extremely low input offset current and high input impedance associated with the FET devices. These characteristics make it especially useful for low drift integrators, sample and hold circuits, and commutating multiplexers.

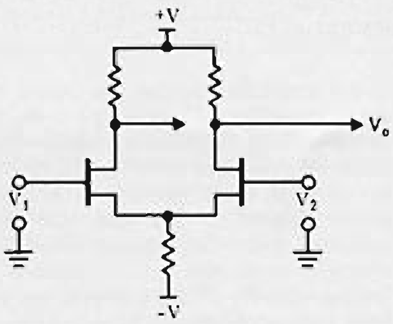


Figure 6 FET Input Stage

Newer types using MOS-FET's achieve input offset current on the order of a picoampere, which makes them attractive for charge amplifiers and electrometer applications. This feature is a trade-off with offset *voltage* and voltage drift.

The *linear integrated circuit* is a differential amplifier formed by monolithic semiconductor technology on a single silicon die. It is similar to the discrete-component differential amplifier except that the user must supply the frequency compensating elements (one or two resistors and capacitors) which are required to complete stability from oscillation. The principal advantage of today's linear IC is *small size*. This means more circuit functions per unit area of pc card, for example, and smaller equipment size for a given degree of complexity. The IC amplifiers have the potential for economy and reliability.

## 2. SPECIFICATIONS

Within all of the amplifier categories described above, a variety of products is available to help the user match the amplifier to his application without buying capability he doesn't need. The table (on the next page) shows the parameters usually specified and the ranges of gain and bandwidth, input performance, and output ratings available with amplifiers now on the market.

Since the operational amplifier is used in a large variety of closed-loop circuits, it is customary to normalize the specifications by using *open-loop* parameters where applicable and to refer closed-loop parameters to the input terminal. The latter process is accomplished simply by dividing the output figures by the closed-loop gain.

### Definition of Terms

There is fairly general agreement in the industry on the definition of terms used to specify the characteristics and ratings of operational amplifiers. Some noteworthy conflicts exist, however, and there may be significant differences in the conditions under which certain specifications apply. The following list should help orient the uninitiated user.

- *DC Voltage Gain, open-loop* is the gain of the amplifier without external feedback. For good performance this parameter should be on the order of 100 times or more greater than the *closed-loop gain* desired in the functional amplifier circuit (= $R_f/R_1$). Under these conditions the closed-loop gain is virtually independent of the open-loop gain. Defining *loop gain* as the ratio of open-loop gain to closed-loop gain, the error by which closed-loop gain is diminished from $R_f/R_1$ is given by 1/loop gain.
- *Gain-Bandwidth Product* is a constant in amplifiers with the usual 6 db/octave *roll-off*, numerically equal to the frequency at which the gain has "rolled off" to unity. An amplifier with a unity-gain *cross-over frequency* of 1 MHz, for example, will have an open-loop gain of $10^4$ at a signal frequency of 100 Hz. In *general-purpose* units the gain is down to unity in the frequency range of 0.5 to 5 MHz. In *wideband* units the unity-gain frequency is in the range of 10 to 100 MHz.
- *Gain Roll-Off* is the rate at which the open-loop gain falls off with increasing signal frequency. The optimum roll-off is 6 db per octave (= 20 db per decade), which is an expression of constant gain-bandwidth product. This is the response associated with a maximum phase shift of —90°, for which the amplifier will be

Continued from page 9

**Comparative Specifications of Currently Available Operational Amplifiers**

| Specifications | General Purpose Differential | Chopper-stabilized | FET-Input | Integrated Circuit |
|---|---|---|---|---|
| DC Voltage Gain, open-loop: | $10^4$-$10^6$ | $10^6$-$10^8$ | $10^4$-$10^6$ | $10^3$-$10^5$ |
| Gain-Bandwidth Product: | 0.4-10MHz | 0.3-10 MHz | 1-30 MHz | 1-10 MHz |
| Slewing Rate Limit: | 0.1-3 V/µs | 0.6-50 V/µs | 0.5-20 V/µs | 0.1-30 V/µs |
| Initial Input Offset: | | | | |
| Voltage: | 0.3-1 mV | 10-200 µV | 1-5 mV | 1-5 mV |
| Current: | 1.5-500 nA | 10-2000 pA | 1-50 pA | 100-500 nA |
| Drift vs. Temperature: | | | | |
| Voltage: | 5-100 µV/°C | 0.5-10 µV/°C | 5-100 µV/°C | 3-50 µV°C |
| Current: | 0.2-5 nA/°C | 0.5-20 pA/°C | 20-100 pA/°C | 0.1-5 nA/°C |
| Drift vs. Time: | | | | |
| Voltage: | 10-100µV/day | 1µV/wk | 5-100µV/day | — |
| Input Noise: | | | | |
| 0.1-1.0 Hz: | — | 5-10 µV p-p | — | — |
| Wideband: | 1-20 µV rms | 10-150µV rms | 2-30µV rms | — |
| Input Impedance, open-loop: | | | | |
| Differential: | 100-500 k | 0.5-1 Meg | $10^{10}$-$10^{12}$ Ω | 10-500 k |
| Common Mode: | 10-500 Meg | — | $10^{10}$-$10^{12}$ Ω | — |
| Input Voltage (maximum): | | | | |
| Differential: | 5-15 V | — | — | 1-8 V |
| Common Mode: | 3-20 V | — | 8-11 V | 1-11 V |
| Common Mode Rejection: | 50-100 db | — | 60-74 db | 60-100 db |
| Output Voltage: | 10-20 V | 10-150 V | 10-11 V | 3-12 V |
| Output Load Current: | 1-20 mA | 2-100 mA | 2-20 mA | 1-10 mA |
| Output Capacitive Loading: | 0.0005-10µf | 0.0005-0.02µf | — | — |

free from any tendency to oscillate under all values of closed-loop gain.

- *Slewing Rate Limit* is the maximum time rate of change of output voltage for a step input. "Step response" is sometimes given as *velocity limit* or as *risetime.* Slewing rate is usually specified in volts per microsecond. It is related to the maximum frequency *f* at which full output voltage *V* can be obtained by $\Delta V/\Delta t = 2\pi fV$.

- *Input Offset Voltage* is the input voltage required to zero the dc component of the output with zero input signal and zero source impedance. Offset is an error inherent to some degree in all practical amplifiers. The offset usually can be adjusted to zero by applying a small voltage derived from the amplifier's power supplies. *Initial* offset signifies that the parameter is determined before any external balance adjustments are made. Offset, drift, and noise (following) combine to add a variable error to the input signal. The total error must be small compared to the minimum signal to be amplified with reasonable accuracy, or linearity.

- *Input Offset Current* is the input current required to zero the dc output current with zero signal and infinite source impedance. Offset current has the effect of an error voltage drop across $R_1$, the series input (or "summing") resistance. In the inverting amplifier the closed-loop *input impedance* (following) is equal to $R_1$; therefore, high input impedance is a trade-off with offset error. In the differential amplifier offset current is the *average* of the currents into the two input terminals with zero output. Most linear IC's and some discrete-component amplifiers identify this parameter as *input bias current*, and specify the *difference* in the two currents as *offset*. Since the differential current may be 10-25% of the average, it is important to note this conflict in terminology. In some applications it is the differential input current that constitutes the error, but in many circuits, especially those involving integration, it is the current into either input that is the limiting factor.

- *Drift vs. Temperature* is the slowly varying change in offset voltage and offset current due to a change in temperature. Voltage drift vs. temperature is usually specified in µV/°C, but it is not necessarily linear over the entire operating temperature range of the amplifier.

- *Drift vs. Time* refers to the similar effect of time on offset voltage and current.

- *Drift vs. Supply Voltage* is a measure of the effect of changes in supply voltages on the offsets. This parameter is properly specified in µV per percent change in supply voltage from the rated value, but it is sometimes given in µV per volt.

- *Input Noise* is the normalized value of any output disturbance not contained in the input signal. Noise is generated in the transistors and resistors of the amplifier, and it may be coupled from power lines and rf sources. To properly specify noise one must define the source resistance $R_1$ as well as the bandwidth. "Wideband" noise voltage includes frequencies up to the range of 1 kHz to 1 MHz, usually specified in rms volts. Some amplifiers are also specified for peak-to-peak noise in the range of 0.1 to 1 Hz.

A basic feature of the differential amplifier is its ability to amplify signals impressed across its input terminals, rejecting signals that appear between the two inputs together and the ground reference. We may define the former as *differential* or *normal mode* signals and the latter as *common mode* signals.

- *Input Impedance, open-loop* is the complex impedance seen looking into the input terminals of the amplifier without external feedback. In the conventional (bipolar transistor)-input differential amplifier $Z_{in}$ depends on the current gain $h_{FE}$ and the intrinsic base resistance $r_e$ of the input devices; it is typically in the range of 200 k to 500 k. The extremely high (typically $10^{11}$ ohms) $Z_{in}$ of the FET-input amplifier, as well, results from the geometry and bulk properties of the FET's. In the chopper-stabilized amplifier, $Z_{in}$ is proportional to the *on* and *off* resistance of the chopper device, typically 1 Meg at dc.

- *Common-Mode Input Impedance* of a differential amplifier is the impedance between the two input terminals together and ground. It is determined largely by the characteristics of the input transistors and the near-infinite impedance of the current source used in the common emitter lead of the input stage.

- *Differential Input Voltage* rating is the maximum voltage that can be applied across the input terminals of a differential amplifier without causing damage to the amplifier.

- *Common-Mode Input Voltage* rating of a differential amplifier is the maximum voltage that can be applied between the two inputs together and ground without causing damage.

- *Common-Mode Rejection* is the ratio of the gain of the amplifier to a differential signal to the gain of the amplifier to a common-mode signal.

- *Output Voltage* rating is the maximum output voltage which the amplifier will develop in the linear operating region; i.e., before the onset of saturation.

- *Output Load Current* rating is the maximum current that the amplifier will deliver to, or accept from, a load. This rating includes the amount, however small, which is caused to flow in the feedback loop.

- *Output Capacitive Loading* is the maximum capacitance that can be placed on the output of the amplifier at unity gain without increasing the phase shift to the point of inducing oscillation. The limiting value increases in direct proportion with the closed-loop gain. The traditional way to accommodate higher loading capacitance is to isolate the load from the amplifier output terminal with a resistance. With some amplifiers the resistance required is only a few ohms, in which case the error and dissipation introduced are minimal.
- *Output Impedance, open-loop* is the complex impedance seen looking into the output terminals of the amplifier with no external feedback. In closed-loop operation the output impedance is equal to the open-loop impedance divided by the loop gain. If the open-loop impedance is not more than a few hundred ohms and the loop gain is high enough for good gain accuracy and stability, the closed-loop impedance will be on the order of an ohm or less, which can be neglected in most applications.

## 3. FEEDBACK AMPLIFIER ANALYSIS

To guide the designer in the use of an operational amplifier to fulfill his requirements, it may be helpful to review just what we achieve with the high intrinsic voltage gain and negative feedback, and how closed-loop performance is affected by the finite open-loop parameters of the practical amplifier.

### Closed-Loop Gain

Let us start by developing an expression for the *transfer function* of a basic feedback amplifier, which we commonly call the *closed-loop gain*. (Fig. 7)

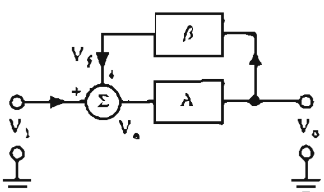


Figure 7 Basic Feedback Amplifier

A is the open-loop gain of our operational amplifier, assumed to be phase inverting so that if $V_1$ is positive, $V_0$ will be negative.

If A were precisely known and controllable, we might satisfy a number of voltage-gain applications without the feedback loop. Usually this is not the case, so we sample the output $V_0$ and feed back a portion β to the input terminal Σ. Here the feedback voltage $V_f$ is summed with the input signal $V_1$. Terminal Σ is defined as the *summing junction*, and since $V_f$ must be a negative number with respect to $V_1$, this terminal is often identified as the *negative* or *inverting* input of the amplifier. β is known as the feedback factor.

The input error voltage is

$$V_e = V_1 + V_f, \qquad (1)$$

and by definition, $V_f = \beta V_0$.

Also,
$$V_e = \frac{V_0}{A}. \qquad (2)$$

Substituting,
$$\frac{V_0}{A} = V_1 + \beta V_0$$

$$V_0 (1-A\beta) = AV_1$$

and
$$\boxed{\frac{V_0}{V_1} = G = \frac{A}{1-A\beta}}, \qquad (3)$$

where G is defined as the closed-loop gain. One sees that if the quantity $A\beta >> 1$,

then
$$\boxed{G \simeq -\frac{1}{\beta}}, \qquad (4)$$

or the *closed-loop gain* is approximately equal to the reciprocal of the feedback factor, which *is independent of the highly variable open-loop gain*. The minus sign denotes phase inversion in the amplifier.

The quantity Aβ is defined as *loop gain*, which involves both the amplifier itself (A) and the external feedback loop used with it (β). It is usually the loop gain that determines how close to the ideal a given amplifier circuit will appear, e.g., whether the simplified relationship of Eq. (4) is valid or not.

*Example:* Find the closed-loop gain of an amplifier with an open-loop gain A = −10,000 if we feed back 10 percent of the output, i.e. β = 0.1.

$$G = \frac{A}{1-A\beta} = \frac{-10\,000}{1+1000} \simeq -9.99$$

Using the same operational amplifier (A = —10 000) but with β = 0.01, we find

$$G = \frac{-10\,000}{1+100} \simeq -99,$$

and again with β = 0.001,

$$G = \frac{-10\,000}{1+10} \simeq -900.$$

Notice that in the above examples $G \simeq -1/\beta$ would have given us —10, —100, and —1000, respectively, and that as the loop gain Aβ becomes smaller, G deviates more and more from $-1/\beta$. Indeed, the gain error is given by $1/(A\beta)$. If $A\beta >> 1$, one sees further from Eq. (3) that

$$\boxed{A\beta \simeq \frac{A}{G}}, \qquad (5)$$

i.e. the loop gain may be determined by the *ratio* of open-loop gain to closed-loop gain.

Returning to Eq. (2) we see that as A approaches infinity, the error voltage $V_e$ approaches zero, i.e., the summing junction is maintained close to the ground reference. With A = $-10^7$ and an output of ±20 volts, for example, $V_e$ will be ±2 microvolts. Using the approximation that $V_e = 0$, which usually is valid, it follows that the error current into (or out of) the amplifier will be zero. The current through the feedback loop, then, is equal to the current through the source.

### Gain Stability

We have stated that the principal effect of the negative feedback loop is to stabilize the voltage gain of the amplifier. This effect can be shown quantitatively by differentiating G with respect to A:

From Eq. (3)
$$G = \frac{A}{1-A\beta},$$

then
$$\frac{dG}{dA} = \frac{1}{(1-A\beta)^2},$$

and
$$\boxed{\frac{dG}{G} = \left(\frac{1}{1-A\beta}\right)\frac{dA}{A}} \qquad (6)$$

Therefore, the relative change in closed-loop gain is approximately equal to the relative change in open-loop gain divided by the loop gain.

*Example:* Find the variation in closed-loop gain using an amplifier with nominal open-loop gain of —10 000 subject to 20 percent deviation, and β = 0.01.

$$\frac{dG}{G} = \left(\frac{1}{1-A\beta}\right)\frac{dA}{A} = \left(\frac{1}{1+100}\right)0.20 \simeq 0.002$$

i.e., G will vary by only 0.2 percent.

### *Where do we Buy a β ?*

The feedback factor β usually takes the form of a precision voltage divider, and the input signal voltage $V_1$ is made to appear as a current source by feeding it through a series resistance: (Fig. 8)

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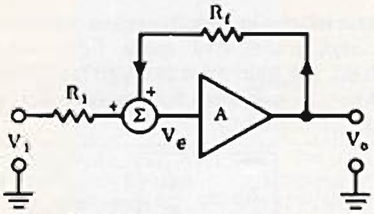



Figure 8 Amplifier With Feedback

By superposition,

$$V_e = \frac{R_f}{R_1 + R_f} V_1 + \frac{R_1}{R_1 + R_f} V_o.$$

Substituting $V_e = V_o/A$,

$$\left(1 - \frac{AR_1}{R_1 + R_f}\right) V_o = \frac{AR_1}{R_1 + R_f} V_1.$$

and,

$$\frac{V_o}{V_1} = G = \frac{\dfrac{AR_f}{R_1 + R_f}}{1 - \dfrac{AR_1}{R_1 + R_f}}$$

Simplifying and factoring out the ratio $-R^f/R_1$,

$$G = -\frac{R_f}{R_1}\left[\frac{1}{1 - \dfrac{R_1 + R_f}{AR_1}}\right] \quad (7)$$

Now Eq. (3) can be manipulated into the form

$$G = -\frac{1}{\beta}\left[\frac{1}{1 - \dfrac{1}{A\beta}}\right] \quad (8)$$

from which, by comparison with Eq. (7), it can be deduced that

$$\frac{AR_1}{R_1 + R_f} = A\beta = \text{loop gain.}$$

Again, if the loop gain is much larger than unity, Eq. (7) reduces to

$$G \simeq -\frac{R_f}{R_1}, \quad (9)$$

which is consistent with Eq. (4) that the closed-loop gain is independent of the open-loop gain. If it is not true that $A\beta >> 1$ we must use the more precise expression, Eq. (7).

*Example:* Find the closed-loop gain of an amplifier with $A = -10\,000$ if $R_1 = 10$ k, $R_f = 10$ Meg. From Eq. (7),

$$G = -\frac{10\,\text{Meg}}{10\,\text{k}}\left[\frac{1}{1 - \dfrac{10\text{ k} + 10\text{ Meg}}{-10\,000\,(10\text{ k})}}\right]$$

$$= 1000\left(\frac{1}{1+0.1}\right)$$

$$= -910$$

Note again that the voltage divider ratio, Eq. (9), would have predicted $G = -1000$. A closed-loop gain of $-1000$ could be achieved, of course, by adjusting $R_f/R_1$ to approximately 1100, but in this example the loop gain is only 10, which is not adequate to insure a high degree of gain stability. If a nominal closed-loop gain of $-1000$ is required, better design practice would be to increase $A\beta$ by selecting another amplifier with higher open-loop gain. With $A = -10^6$, for example, the loop gain is increased to 1000 and the error due to finite open-loop gain is reduced to 0.1 percent.

**Frequency Response**

It is important to realize that the high open-loop gain we have been using is available only at dc and very low frequencies. At higher frequencies the gain is attenuated markedly, due largely to the effects of transistor output capacitance. To insure stable operation, the operational amplifier usually is compensated to provide a smooth roll-off of gain with increasing frequency.

The amplifier therefore looks like an RC lag network which attenuates the dc open-loop gain A; the effect of frequency on A may be seen by analyzing the response of this network: (Fig. 9)

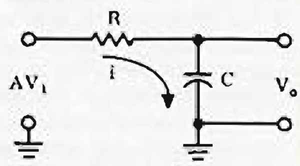



Figure 9 RC Lag Network

The current i through R and C is

$$i = \frac{AV_1}{R + X_c} = \frac{AV_1}{R + \dfrac{1}{j\omega C}}$$

where $\omega = 2\pi f$ is the frequency in radians, and j is the complex operator. Therefore,

$$V_o = \frac{i}{j\omega C} = \frac{AV_1}{1 + j\omega CR}$$

and

$$\frac{V_o}{V_1} = A(\omega) = 1 + \frac{A}{j\omega CR} \quad (10)$$

where $A(\omega)$ is the open-loop gain as a function of frequency.

The frequency response of a lag network is shown in Figure 10.

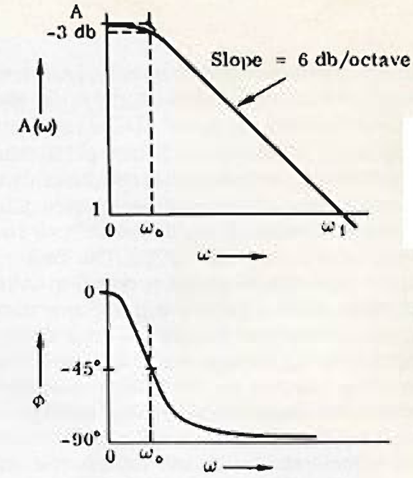



Figure 10 Response of RC Network

This display of gain and phase shift vs. frequency is known as a Bode plot.

The break frequency $\omega_o$ occurs when $X_c = R$; then $\omega_o = 1/(RC)$ and

$$A(\omega) = \frac{A}{1 + j\dfrac{\omega}{\omega_o}} \quad (11)$$

The magnitude of $A(\omega)$ then is

$$A(\omega) = \frac{A}{\left[1 + \left(\dfrac{\omega}{\omega_o}\right)^2\right]^{1/2}} \quad (12)$$

and the phase shift $\varnothing$ is

$$\varnothing = -\tan^{-1}\frac{\omega}{\omega_o} \quad (13)$$

That $A(\omega)$ is down 3 db from A at the break frequency may be shown by evaluating Eq. (12) at $\omega = \omega_o$: Then

$$\frac{A(\omega)}{A} = \frac{1}{(1+1^2)^{1/2}} = 0.71 = -3\text{ db.}$$

Also,

$$\varnothing_o = -\tan^{-1} 1 = -45°.$$

In chopper-stabilized amplifiers the principal break frequency is less than 1 Hz, therefore it is of interest to examine Eq. (12) at $\omega >> \omega_o$. This yields

$$\frac{A(\omega)}{A} = \frac{\omega_o}{\omega},$$

or

$$A(\omega)\,\omega = A\,\omega_o \simeq \text{constant.}$$

Since $\omega_o \simeq 0$ we may interpret $\omega$ as *bandwidth*. The practical amplifier is characterized by a reasonably constant *gain-bandwidth product*. This figure is numerically the same, then, as the unity-gain cross-over frequency $\omega_1$ at which $A(\omega) = -1$,

i.e.,
$$A(\omega)\,\omega = -\omega_1 \quad (14)$$

The nearly constant slope of the response may be described as a *gain roll-off* of 6 db per octave, or 20 db per decade. This is simply a statement that the gain is down by a factor of 2 when the frequency doubles, or the gain is down by 10X if the frequency is increased by 10X.

*Example:* Find the closed-loop gain of an amplifier with a dc open-loop gain, $A = -10^7$, gain-bandwidth product $f_1 = 1$ MHz, if $R_1 = 10k$, $R_f = 1$ Meg, and the signal frequency f = 100 Hz.

From Eq. (14) $A(\omega) = -\dfrac{1 \text{ MHz}}{100 \text{ Hz}} = -10^4.$

Now from Eq. (7), using $A(\omega)$ instead of A,

$$G = -\frac{R_f}{R_1}\left[\frac{1}{1-\dfrac{R_1+R_f}{A(\omega)\,R_1}}\right]$$

$$= -\frac{1 \text{ Meg}}{10k}\left[\frac{1}{1-\dfrac{10k+1 \text{ Meg}}{-10^4\,(10k)}}\right]$$

$$= -100\left(\frac{1}{1+0.01}\right)$$

$$= -99.$$

The important point here is that *at a frequency of 100 Hz, the applicable open-loop gain is* $A(\omega)$ and not the dc open-loop gain given on the amplifier specification sheet.

Let us put this example on the Bode plot, as shown in figure 11.

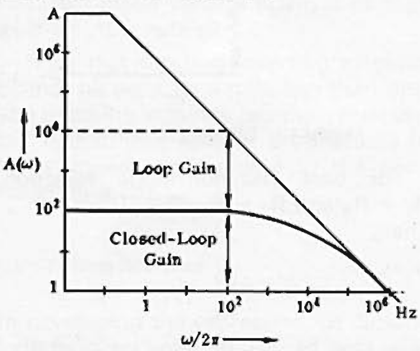



Figure 11 Open and Closed Loop Response

Since gain is plotted on a logarithmic scale (db), the loop gain (= open-loop gain/closed-loop gain) appears as the difference between the open-loop gain $A(\omega)$ and the closed-loop gain. The loop gain $A(\omega)\beta = 100$, and the closed-loop gain G is approximately 100X with an error of 1/100.

Note that the closed-loop response is flat until it approaches the roll-off, whereupon it merges with the open-loop response. Beyond this point the effect of the negative feedback loop is no longer present.

Certain amplifiers are compensated with a higher break frequency to extend the usable gain-bandwidth capabilities of the amplifier to higher signal frequencies.

The unity gain-bandwidth of the amplifier is relatively fixed, however, so the gain must roll off at a faster rate, e.g. at 12 db/octave. This rate of closure with the closed-loop gain line is accompanied by additional phase shift, unfortunately, with the result that the amplifier is no longer stable under all conditions of closed-loop operation.

Another consequence of the attenuation of $A(\omega)$ is increasing input error voltage, $V_e$. Recall that an amplifier with $A = -10^7$ and $V_o = \pm 20$ volts, for example, would have an input error of $\pm 2$ microvolts at dc. If the signal frequency is 100 Hz, however, and $A(\omega) = -10^4$, then $V_e$ would be $\pm 2$ millivolts.

### Transient Response

The closed-loop response of an amplifier to a pulse input or step function is an exponential with the operational time constant. The time required to reach a given proportion of the final output voltage is proportional to the closed-loop gain and inversely proportional to the unity-gain bandwidth.

The inherent bandwidth of the amplifier, then, along with the roll-off compensation, affects the basic step response of the amplifier. This response is usually specified as *slewing rate limit* $\Delta V/\Delta t$, or maximum rate of change of output voltage, in volts per microsecond.

Occasionally this response is specified as the maximum frequency $f_m$ at which the peak output voltage $V_m$ can be realized:

$$\frac{\Delta V}{\Delta t} = 2\,\pi f_m\, V_m,$$

then

$$f_m = \frac{\Delta V/\Delta t}{2\pi V_m}.$$

For sine-wave signals the limiting factor is usually not slewing rate, but gain-bandwidth product and the available loop gain.

### Input and Output Impedance

The infinite input impedance and zero output impedance of the ideal amplifier are approached in the real world as shown in the table of comparative specifications. Open-loop input impedance ranges from about 200 k to $10^{12}$ ohms, output impedance from a few ohms to a few thousand ohms.

Finite input impedance $Z_{in}$ does not have a direct effect on closed-loop gain, but if the summing resistance $R_1$ is comparable to or greater than $Z_{in}$, gain accuracy is degraded through a reduction in the loop gain. Finite output impedance $Z_o$ has a similar effect, but usually negligible if $Z_o$ is less than a few hundred ohms.

With feedback, the open-loop figures change considerably. In the basic inverting amplifier, the closed-loop input impedance is substantially the summing resistance $R_1$. In the non-inverting connection of the differential amplifier, the closed-loop input impedance is equal to the open-loop impedance times the loop gain. This effective high-impedance feature of the differential amplifier is often utilized to accommodate high-impedance sources without loading.

Closed-loop output impedance is equal to the open-loop impedance *divided* by the loop gain, which can yield a very small value. With a loop gain of 1000, or higher, therefore, it matters little whether the open-loop output impedance is 5 ohms or 500 ohms.

## 4. APPLICATIONS

### Typical Circuits

The endless collection of circuits which make effective and convenient use of the operational amplifier attest to its versatility. Many examples are described in the technical and trade literature, and a full recounting is beyond the scope of this paper.

Some of the more frequently used operational amplifier circuits are shown below.

### Summing Amplifier

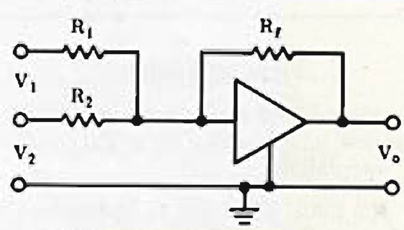



Figure 12 Summing Amplifier

This circuit illustrates summing and inverting voltage gain.

$$V_o = -R_f\left(\frac{V_1}{R_1} + \frac{V_2}{R_2}\right).$$

Continued from page 13

where the negative sign indicates phase inversion of $V_O$ with respect to the input.

Note that if $V_2 = 0$, then $V_O = -\frac{R_f}{R_1} V_1$

the closed-loop input impedance is $R_1$ for source $V_1$.

#### Inverting Gain, High Impedance

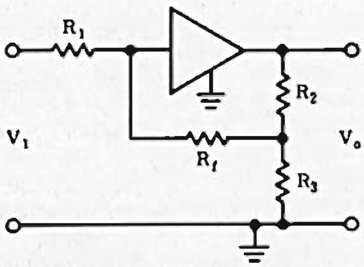



Figure 13 Inverting, High Impedance Amplifier

In this circuit $V_O$ is divided and then a portion fed back to provide high closed-loop gain and high input impedance (= $R_1$) without the need for excessively high feedback resistance.

$$V_o \simeq -\frac{R_2}{R_3}\left(\frac{R_f}{R_1} V_1\right)$$

If $V_1$ is a reference voltage source and $R_2$ is a floating load, this circuit acts as a constant current supply, delivering large output current $I_O$ with low current drain from the reference source.

$$I_O = \frac{V_{ref}}{R_1}\left(\frac{R_f + R_3}{R_3}\right)$$

#### Integrator

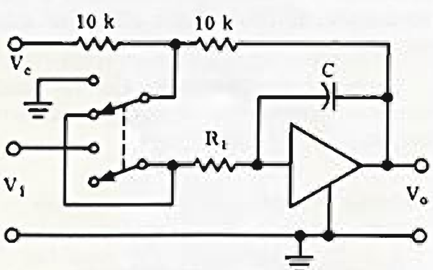



Figure 14 Integrator

Here an initial reference state must be defined, which shows up as the constant of integration.

The initial condition is established by charging C to $-V_c$; then $V_1$ is switched in to integrate.

$$V_o = \frac{-1}{R_1 C}\int_0^t V_1\,dt - V_c$$

If $V_c = 0$, $R_1 = 100\,k$, $C = 1\,\mu f$, then $V_O = -10\int_0^t V_1\,dt$. The integration ramp will continue until $V_O$ reaches the saturation level of the amplifier or until it is otherwise terminated. The method of switching from *initial condition* to *integrate* isolates the summing junction from the disturbing effects of switching transients.

To measure integrator drift, $\Delta V/\Delta t$, one may set the initial $V_c$, then set $V_1 = 0$ and observe any deviation from the initial offset, $\frac{\Delta V}{\Delta t} = \frac{i}{C}$, where i is the input offset current of the amplifier. In practical applications the integration time *t* may vary from a few milliseconds to several hours so the limit of acceptable input current will vary accordingly.

#### Voltage Comparator

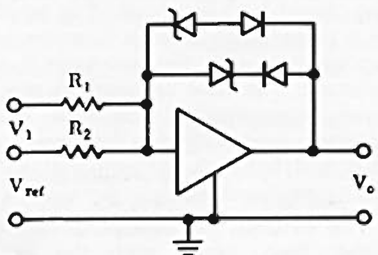



Figure 15 Voltage Comparator

$V_{ref}$ is of opposite polarity to the unknown $V_1$, and $R_1$, $R_2$ provide scaling if necessary. $V_O$ detects the coincidence of $V_1$ to the absolute value of $V_{ref}$.

#### Overload Clamps

Excessive input voltage will saturate the amplifier in the simple voltage-gain applications. Since recovery from the overload in a chopper-stabilized amplifier may take several seconds after removal of the offending signal, it is often prudent to prevent saturation. In simplest form the output may be clamped with a pair of back-to-back Zeners across the feedback resistor, limiting $V_O$ to $\pm V_z$, the Zener voltage. During normal operation the amplifier is in the linear mode and the gain is determined by $-R_f/R_1$. But when the $V_O$ reaches $V_z$ the Zeners conduct, increasing the negative feedback and preventing $V_1$ from driving $V_O$ into saturation. Recovery time is reduced to milliseconds, but one must be aware that any leakage current through the diodes can introduce significant error.

The following circuit illustrates an improved clamp which reduces the leakage problem.

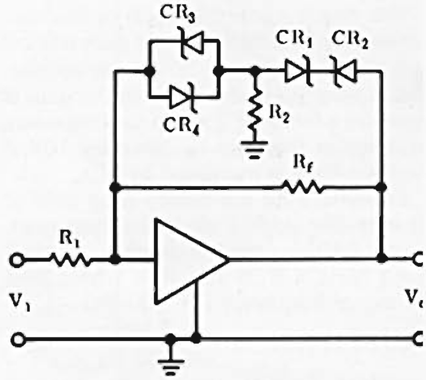



Figure 16 Overload Clamp Circuit

In normal amplifier operation $R_2$ ab sorbs the leakage of the Zeners $CR_1$ an $CR_2$; the minute drop across $R_2$ cause only a very small leakage through diod $CR_3$ and $CR_4$ into the summing junctio

#### Differential Voltage Gain

This circuit shows the basic invertin amplifier connection for a differenti input, single-ended output:

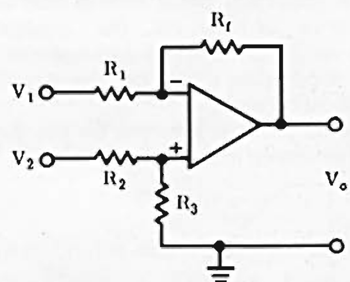



Figure 17 Differential Amplifier

For best common-mode rejectio $R_1 = R_2$ and $R_3 = R_f$.
Then,

$$V_O = -\frac{R_f}{R_1}(V_1 - V_2)$$

#### Non-Inverting Gain

The differential amplifier may be connec ed in the following manner to deliver a output voltage in phase with the inpu

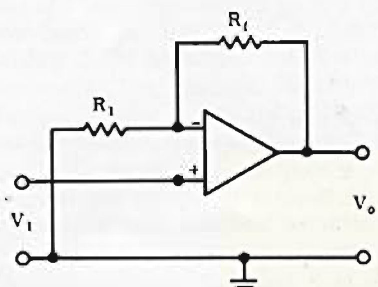



Figure 18 Non-Inverting With Gain

$$V_o = +\left(\frac{R_f + R_1}{R_1}\right) V_1$$

An important feature of this circuit is that it maintains high input impedance even with low values of $R_1$ and $R_f$. (Closed-loop $Z_{in}$ = open-loop $Z_{in}$ times the loop gain.)

If we remove $R_1$, we have the unity-gain voltage follower, often used as an isolation buffer. Here virtually no current flows into the summing junction, hence through the feedback loop. Theoretically, therefore, $R_f$ may have any finite value, but offset current errors and the effects of stray capacitance are minimized with $R_f = 0$.

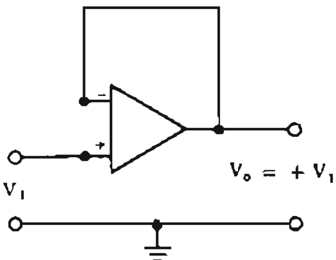



Figure 19 Voltage Follower

Since $V_1$ is the *common-mode input voltage*, the full output voltage capability of the amplifier can be utilized only if the common-mode voltage rating is as high as the output rating.

Note that the non-inverting amplifier cannot be used for a gain less than one. The inverting amplifier has this versatility, but it does not achieve the increase in input impedance possible in the non-inverting mode.

#### Gain-Setting Resistors

In developing the expression for closed-loop gain we were concerned only with the ratio of $R_f$ to $R_1$. The magnitude of these resistors is not completely arbitrary, and the following considerations may aid the selection process.

First, since input current offset, drift, and noise are proportional to $R_1$, this resistance should be kept reasonably small. The optimum range of $R_1$ lies between the bounds for amplifier specification compliance and source loading.

The feedback resistor $R_f$ should be low enough to draw a current of at least one microampere ($= V_o/R_f$) in order to firmly establish the voltage divider. In addition, The stray capacitance of very high resistance values tends to degrade the frequency response. On the other hand, $R_f$ must be large enough to produce the desired gain with a reasonable value of $R_1$.

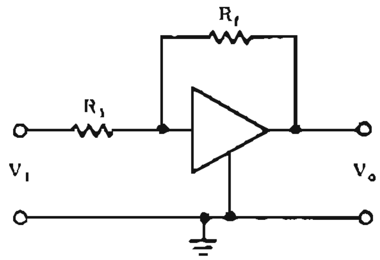



Figure 20 Gain Setting Resistors

With these considerations observed, it is usually possible to adjust $R_1$ and $R_f$ to standard values that will yield the desired gain. In general it is feasible to use resistors between about 1 k and 20 Meg to achieve closed-loop gains up to about 1000X. Usually metal-film resistors are necessary in order to preserve the temperature and time stability of the amplifier.

#### Amplifier Selection

The first decision to make in selecting an amplifier is whether a *single-ended* or a *differential* input is required. For operation in the normal linear mode this is often a question of the physical disposition of the source and load or other factors affecting the compatibility of grounds. If the load is isolated from ground, a differential output is required as well.

Secondly, we must consider the *voltage level and impedance of the source* in relation to the offset, drift, and noise specifications. It may be feasible to balance out the initial offsets, but the drift and noise of the amplifier, of course, must be low enough to permit resolution of the minimum signal level. In some applications, such as those involving integration, the input offset current is the determining parameter.

Next, we need to select an amplifier with an open-loop gain that is high enough for the application. "High enough for the application" means that the resulting *loop gain* (open-loop gain/closed-loop gain) will yield satisfactory stability and gain accuracy. Remember that in calculating loop gain the specified dc open-loop gain may be used only for dc and very low frequency signals. At frequencies above a few Hz the effective open-loop gain is found by dividing the specified gain-bandwidth product by the signal frequency.

In certain applications it is necessary to match the output capability of the amplifier to the voltage and current required to drive the load. In critical applications the input specifications dominate the choice, since the output can be boosted as required with a fairly simple buffer.

Other considerations include package size and style, operating temperature and compatibility with other environmental conditions, power supply constraints, and cost. The relative importance of these items depends on the requirements of the whole system concerned as well as the specific function of the amplifier.

by C.V. Weden, Fairchild Instrumentation

## Capacitor improves sample-and-hold circuit

J. N. Giles, *Fairchild Semiconductor, Mountain View, Calif.*

Fig. 1a.

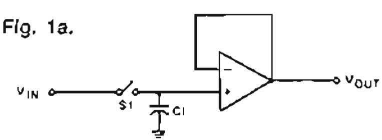
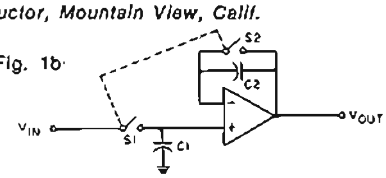



Fig. 1b.




Marked improvement in voltage-holding ability of a sample-and-hold circuit is possible when a capacitor is added (b) to the conventional circuit (a).

Conventional sample-and-hold circuits using operational amplifiers have the general form of Fig. 1a. The voltage to be held is sampled through switch *S1* and stored on capacitor *C1*. The amplifier functions as a high-input-impedance, unity-gain buffer between the voltage on the capacitor and the outside world. The charge on the storage capacitor leaks off at a rate determined by the amplifier input bias current and the shunt resistance to ground.

The addition of capacitor *C2*, equal to *C1*, between the output and the inverting input of the amplifier (see Fig. 1b) improves the decay time of the circuit by better than a factor of ten. The circuit operates as before, except that leakage across *C1* is now compensated for by an equivalent leakage across *C2* such that the output voltage remains almost constant, depending on the degree of match between the two input bias currents and the capacitors. The output drift can even be adjusted to zero by trimming one of the capacitors to compensate for the small difference in bias currents.

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