

# Graphs aid selection of a-d converters

by Raymond J. Tarver  
Raytheon Co., Equipment Division, Wayland, Mass.

Although analog-to-digital converters are widely used circuit components these days, they are frequently not specified properly by designers. In addition to the correct resolution, accuracy, speed, and temperature stability, a-d converters must be able to provide a given system dynamic range or signal-to-noise ratio.

Too often, designers neglect to take into consideration how converter quantization noise relates to other system noises. The result is a poor effective dynamic range or signal-to-noise ratio. The graphs given here make it easier to pick the right converter for the job.

For an ideal system, one that has no internal or external noise sources, and one in which the required variations on the signal are actually part of the signal, the signal-to-quantization noise power ratio is:

$$(SNR)_a = 12[S(t)]^2/Q^2 \quad (1)$$

where  $S(t)$  is the signal, and  $Q$  is the quantization increment. This latter variable is given by:

$$Q = R/N = R/(2^m - 1) \quad (2)$$

where  $R$  is the range or maximum magnitude of the signal being quantized,  $N$  is the number of available discrete quantization levels, and  $m$  is the number of bits (including the sign bit) provided by the converter.

In the real world, Eq. 1. is equivalent to defining any additive noise as part of the signal, or having a signal with noise-like variations. The signal-to-noise ratio of a real system having internal and external additive noise is given by:

$$SNR = [S(t)]^2 / ([N_i(t)]^2 + [N_a(t)]^2 + [N_q(t)]^2) \quad (3)$$

where  $N_i(t)$  is the input noise,  $N_a(t)$  is the internal noise, and  $N_q(t)$  is the a-d quantization noise. This latter quantity can be expressed as:

$$N_q(t) = Q / \sqrt{12}$$

Naturally, the quantization noise can be made arbitrarily small by adding more bits to the a-d converter, although practical limitations, such as cost and availability, often limit the number of bits. In any event, if  $N_q(t)$  is reduced to the point where  $N_a(t)$  and/or  $N_i(t)$  dominates the signal-to-noise ratio, obviously there is little reward in decreasing  $N_q(t)$  further. This is another

practical limitation on the number of converter bits chosen for a particular application.

Furthermore, cost and availability also enter in the reduction of  $N_i(t)$  and  $N_a(t)$ . Hence, there must be a trade-off between the three noise sources. In high-data-rate radar applications, the remainder of the system is often designed around what value of  $N_q(t)$  can be achieved with reasonable risk.

Equation 3 can be rewritten as:

$$SNR = S^2(t) / ([N_i(t)]^2 + [N_q(t)]^2)$$

where:

$$[N_t(t)]^2 = [N_i(t)]^2 + [N_a(t)]^2$$

Let:

$$N_t(t) = kN_q(t)$$

then, for values of  $k$  greater than or equal to 0:

$$SNR = S^2(t) / (k^2 + 1)[N_q(t)]^2 \quad (4)$$

where  $k$  represents the ratio of the root-mean-square value of fixed noise to the rms value of quantization noise:

$$k = \frac{\text{rms fixed noise}}{\text{rms quantization noise}}$$

Equation 4 can be further simplified by normalizing the signal,  $S(t)$ , to unit range ( $R$ ):

$$SNR = 12 / (k^2 + 1)Q^2 \quad (5)$$

Substituting Eq. 2 in this last equation yields:

$$SNR = 12(2^m - 1)^2 / (k^2 + 1) \quad (6)$$

Graph 1 is a plot of Eq. 6 with  $k$  as a parameter. As the nomograph shows, increasing values of  $k$  mean that more converter bits are needed to preserve a system's signal-to-noise ratio or dynamic range.

If dynamic range is defined as the ratio of the peak signal to the rms noise level, then Eqs. 5 and 6 also define the dynamic range as a function of the number of bits of quantization for a linear unipolar signal. For a bipolar signal, Eq. 6 is high by a factor of two, since half the range is expended quantizing the opposite polarity.

Graph 2 is a normalized plot of Eq. 6 that shows the degradation in dynamic range (or signal-to-noise ratio) as  $k$  departs from its ideal value of  $k = 0$ . At about  $k = 1$ , which corresponds to the knee of the curve, the dynamic range starts to deteriorate rapidly. □

## BIBLIOGRAPHY

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