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A STUDY OF NOISE IN VACUUM TUBES AND ATTACHED CIRCUITS*

By

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Summary—*The noises originating in vacuum tubes and the attached circuits are investigated theoretically and experimentally under three headings: (1) shot effect with space charge, (2) thermal agitation of electricity in conductors, (3) noise from ions and secondary electrons produced within the tube.*

A theoretical explanation of the shot effect in the presence of space charge is given which agrees with experiment insofar as a direct determination is possible. It is shown that the tubes used should be capable of operating at full temperature saturation of the filament in order to reduce the shot effect.

In the computation of the thermal noise originating on the plate side of a vacuum tube, the internal plate resistance of the tube is to be regarded as having the same temperature as the filament.

Noise produced by ions within the tube increases as the grid is made more negative.

With tubes properly designed to operate at temperature saturation it is possible to reduce the noise on the plate side to such an extent that the high impedance circuits employed on the grid side of the first tube of a high gain receiving system contribute practically all of the noise by virtue of the thermal agitation phenomenon.

Introductory

It has long been realized that there is a limit to the amount of amplification which may usefully be employed in a circuit containing vacuum tubes, and that in the absence of static this limit depends upon noises which arise in the circuit itself. Of these noises, those which come from run-down batteries, poor connections, vibration, and microphonic effects, either within the vacuum tubes or in some part of the external circuit, will not be discussed here since their remedy, although not always easy, is obvious. With these eliminated from consideration, there remain several distinct ways in which noise occurs in the circuit.

In the first way noise is produced by irregularities in the stream of electrons from the filament to the plate of the vacuum tube. In the absence of space charge this noise has been termed by Schottky the "schroteffekt," or "small shot effect," from the analogy which the flight of electrons from the filament to the plate of a vacuum tube bears to the spattering of small shot fired from a shot gun. The simple term "shot effect" will be used in this paper to denote this noise either with or without space charge.

* Dewey decimal classification: R170.

In the second way noise is produced by the thermal agitation of electric charges within the conductors of the circuit. This noise is discussed from a theoretical viewpoint in a paper by Dr. H. Nyquist[1] and from an experimental standpoint in a paper by its discoverer, Dr. J. B. Johnson.[2] In Part II of the present paper the importance of this noise, which will be termed "thermal noise," in high-frequency radio receiving circuit design will be discussed.

In still other ways, noise may be produced by agencies operating within the tube, such as the ionization of gas molecules, the production of secondary electrons and ions by bombardment of the grid or the plate, and by evaporation of ions from the filament. These and similar agencies may conveniently be grouped in a third general source of noise within vacuum tubes and their associated circuits, which will be discussed in Part III.

Part I

SHOT EFFECT

For the theory of the shot effect in the absence of space charge the reader is referred to a paper by Dr. T. C. Fry.[3] A partial picture of its mechanism may be obtained by considering what happens when a single electron is transferred from the filament to the plate of the vacuum tube. Since the time of flight of the electron is brief in comparison with the periods even of the highest of the frequencies with which we have at present to deal, the effect of the electron on the circuit is equivalent to that of suddenly placing a charge, ϵ, equal to the charge of the electron, on the plate. This charge is dissipated in the circuit, producing current in the ordinary manner. The total space current is the resultant of all the currents produced individually by the electrons as they arrive at the plate.

If the electrons were to arrive in a uniform stream, that is, if the time between successive electron arrivals were a constant, then the resulting space current could be represented by a Fourier's series, of which the constant term would represent the average value of the space current. Moreover, the term representing the fundamental frequency would have a period equal to the time between successive electron arrivals so that *the lowest frequency present in the resulting current would be that corresponding to the total number of electrons which arrive per second.* Such a frequency is far greater than any to which present day radio apparatus is responsive. It is, therefore, apparent that in

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[1] "Thermal agitation of electric charge in conductors," *Phys. Rev.*, **32,** (110) 1928.

[2] "Thermal agitation of electricity in conductors," *Phys. Rev.*, **32,** (97), 1928.

[3] "The theory of the schroteffekt," *Jour. of the Franklin Inst.*, **199,** No. 2 February, 1925.

order for irregularities to become manifest, it is necessary that they occur in the rate of arrival of the electrons. The stream constituting the space current may be pictured as a moving fluid, of gaseous nature, but of varying density. The effect of the variations on the measuring or the recording device, which in speech receivers is ultimately the human ear, is limited to those frequencies to which the complete system, including the measuring device, is responsive.

The theory of the shot effect has been investigated experimentally by several workers.[4,5] This theory is not, however, directly applicable to radio tube circuit use. The theory is based upon lack of space charge, and only so long as a vacuum tube is operated under such conditions can the noise be thus computed.

In practice the vacuum tube requires the presence of space charge in order to function properly as an amplifier or a detector. Under these conditions Fry's formula does not hold. Johnson showed that as the filament current is increased from zero the noise at first increases rapidly as predicted by Fry's formula; next, however, it goes through a maximum as the space charge comes into play, and then decreases to a value which is nearly independent of filament current. It is in the last named region that vacuum tubes are usually operated.

To see why the noise decreases in the presence of space charge it is necessary to review the assumptions underlying Fry's formula. First, it is assumed that the electrons are emitted from the filament independently of one another. By means of this assumption the effect of the variation in the rate of electron emission can be calculated. Secondly, it is assumed that all electrons emitted by the filament are drawn over to the plate, and consequently that variations in the filament emission are transferred to the plate without change.

The presence of space charge obviously does not affect the first assumption. The filament continues to emit electrons in a manner dependent upon its temperature. The second assumption, however, does not hold when space charge comes into play. In fact, if a curve be drawn showing the relation between filament temperature and rate of arrival of electrons on the plate (see Fig. 1) it becomes evident that at the higher temperatures a lower percentage of the emitted electrons reach the plate. To the right of the point A in Fig. 1 a change in filament emission, resulting from a change in filament temperature, produces no change in the rate of arrival of electrons at the plate. Since the shot effect has been shown to result from changes in the den-

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[4] J. B. Johnson, "The Schottky effect in low frequency circuits," *Phys. Rev.* **26**, No. 1, July, 1925.

[5] A. W. Hull and N. H. Williams, *Science*, p. 100, Aug. 1, 1924; *Phys., Rev.*, **25**, 147, February, 1925.

sity of the electron stream from filament to plate it may be inferred that when the filament has reached full temperature saturation, as shown by the flat portion of the curve to the right of the point A then small fluctuations in the density of the stream emitted by the filament are all smoothed out in the space charge region, and the current reaching the plate contains no variations which result from changes in the filament emission. *Shot effect, as such, is zero in the region of temperature saturation of the filament.*

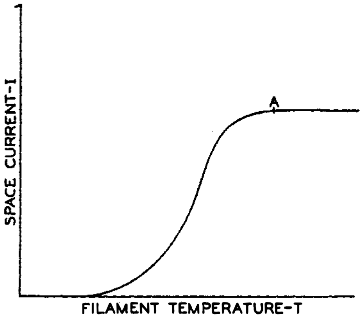

Fig. 1—Variation of space current with filament temperature.

The mathematical formulation for this effect, together with computation formulas for the shot effect at various degrees of partial temperature saturation is given in Notes 1 and 2, to be found at the end of the present paper. Difficulties inherent in obtaining complete experimental verification are discussed in Part IV.

The first requirement for a noiseless circuit is thus seen to be that the vacuum tubes employed contain filaments capable of operating at full temperature saturation, so that the shot effect noise is reduced to zero. The remaining noises must then come from ionization of gas within the tube, from the production of secondary electrons or ions, or from thermal agitation of electricity. This latter effect predominates in high vacuum tubes, and is discussed in the following paragraphs.

Part II

THERMAL NOISE

The noise which arises from the thermal agitation of electricity in conductors is of such recent discovery, and may prove to have such important consequences that a general description of it is given here, even though it entails some repetition of what already has been published by Drs. Johnson[2] and Nyquist.[1] The phenomenon is described in the words of Johnson as follows:

> "The electric charges in a conductor are found to be in a state of thermal agitation, in thermodynamic equilibrium with the heat motion of the atoms of the conductor. The manifestation of the phenomenon is a fluctuation of potential difference between the terminals of the conductor which can be measured by suitable instruments."

In radio-frequency receiving systems a tuned input circuit is usually connected to the grid of the first vacuum tube. Together with the heat motion of the atoms within the conductors which compose this circuit, the electric charges within the conductors are in a state of thermal agitation.* This agitation causes energy to be transferred to and fro between the various parts of the circuit. Although the resultant energy is always a constant, the haphazard surging of charges causes small varying potential differences to appear between any two points located on the conductors of the circuit. These potential variations are amplified and cause a dissipation of energy in a receiving or measuring device located at the terminus of the amplifier. The frequencies present in the measuring device are determined by the frequency characteristics of the receiving system in a way analogous to that in which the frequencies characteristic of shot effect or of long range static depend upon the circuits in which the effects act rather than upon frequencies inherent in the effects themselves. The thermal agitation of charges in conductors will then yield a steady hiss type of sound similar to long range static and to the shot effect.

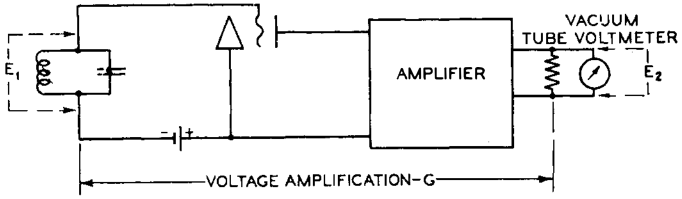

Fig. 2—Circuit to illustrate method of detecting or measuring noise from the thermal agitation of electricity in conductors.

In the system shown in Fig. 2, varying potential differences across the grid of the first tube are produced by thermal agitation of charges within the conductors of the input circuit. These varying potential differences are amplified by the system and produce an energy dissipation in the measuring device or receiver at the end of the diagram. The mean square voltage across the measuring device is given by the expression, developed by Nyquist:

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* Whether or not Maxwellian equilibrium exists between the motions of atoms and electrons, the resulting voltage fluctuations are calculable from the thermal agitations of the atoms alone, at the atomic temperature.

$$\overline{E^2} = \frac{2}{\pi} kT \int_0^\infty R \mid G \mid^2 d\omega \qquad (1)$$

where

k = Boltzmann's constant
= 1.372×10^{-23} joules per degree
T = temperature, degrees Kelvin
R = resistance component of the impedance measured across the input circuit.
G = gain of the amplifier attached to the circuit, i.e., the ratio of E_2 to E_1 (see Fig. 2).

This expression shows that, with a given band width in the amplifier, the effect depends only upon the *resistive component* of the input circuit impedance and upon the *absolute temperature* of that circuit. Thus, if all other noises in the amplifier were eliminated, this effect would represent a limit beyond which further reduction of noise is impossible, since the material composing the input circuit has absolutely no influence either upon the character or magnitude of the noise.

For amplifiers employed in radio, it is possible to design tubes such that the level of the noise produced by the high-impedance circuit on the grid side is raised by the amplification of the tube to a point where it masks the noise produced on the plate side. However, it is important to investigate the noise on the plate side in order to determine the amplification necessary to bring about this result, and also in order to deal with those special cases where it may be desirable to operate the high gain amplifier from a low-impedance input circuit. Suppose, then, that the system shown in Fig. 2 is operated with the grid of the first tube effectively short-circuited to ground for the radio frequencies. Thermal noise from the grid side of that tube is thus eliminated. Also suppose that the filament of the first tube operates at full temperature saturation so that there is no shot effect noise. Experiment shows that under these circumstances a considerable amount of noise still arises in the first tube and its circuits. This residual noise is mostly thermal noise from the resistive component of the impedance measured across from plate to ground of the first tube. This impedance includes the internal resistance of the vacuum tube, and brings forth the following question:

What is the effective temperature as regards producton of thermal noise of the internal plate resistance of a vacuum tube?

This question is discussed in Note 3, where it is shown that the plate resistance must be taken at the filament, or cathode, temperature. Since this temperature ranges between 1000 degrees Kelvin and 2000 degrees Kelvin for the tubes in general use today, it is evident that a fairly large portion of the noise may be contributed by the plate resistance. Equation (1) gives the mean square value of the thermal e.m.f. resulting from a single resistance. From this it is easy to deduce, (see Note 4), that the mean square e.m.f. resulting from the plate resistance at filament temperature in parallel with an external impedance at room temperature T_0 and having a phase angle whose tangent is ϕ, is given by:

$$\overline{E}\ \frac{2kT_0}{\pi}\int_0^\infty R\left(\frac{ab(1+\phi^2)+1}{(1+b)^2+b^2\phi^2}\right)|\,G\,|^2 d\omega \tag{2}$$

where

R = resistive component of external impedance.
b = ratio of external resistance R to internal tube resistance, r_p.
a = ratio of cathode temperature to room temperature.
T_0 = room temperature, Kelvin.
$\phi = X/R$, where X is the reactive component of the external impedance.
G = gain of that portion of the amplifier which follows the tube under measurement.

For the important case where the external impedance is resistive, only, so that $\phi=0$, equation (2) may be written in the simpler form:

$$\overline{E^2}=\frac{2kT_0}{\pi}\int_0^\infty R\left(\frac{ab+1}{(1+b)^2}\right)|\,G\,|^2 d\omega. \tag{3}$$

The form of this equation for the thermal noise from a tube is shown graphicallyby the solid line curves of Fig. 3, where the values of $b = R/r_p$

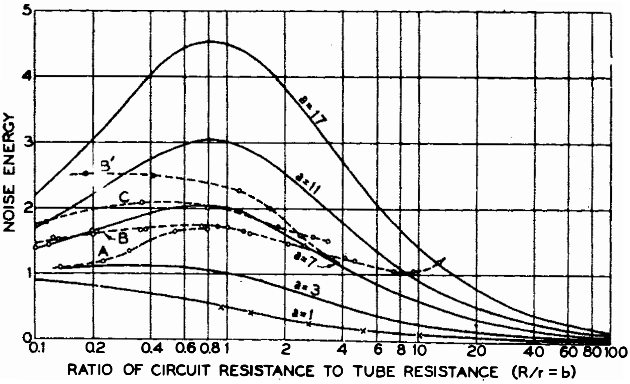

Fig. 3—Noise from thermal agitation in the plate circuit when the plate resistance is varied. The various curves are for different values of filament temperature.

$$\text{Noise Energy } \alpha \frac{1+ab}{(1+b)^2}$$

where a = ratio of filament to room temperature, $b = R/r_p$ = ratio of circuit resistance to tube resistance.
The dotted curves are from experimental data.

are plotted as abscissas, and of noise energy as ordinates. A separate curve is shown for each of several values of filament temperature ranging from room temperature to seventeen times that temperature. Ordinary tubes should show thermal noise values lying between the isothermals for three and seven times room temperature.

Actually, as shown by experiments described in Part IV, and illustrated by the dotted curves on Fig. 3, the tubes gave values lying considerably above the computed curves, even when it was ascertained that the contribution of shot effect, as described in Part I, was negligible. This residual noise is ascribed to ions and to the emission of secondaries from the grid, screen, or plate, as a result of bombardment by the primary electrons. Part III is devoted to a discussion of such effects.

Part III

NOISE PRODUCED BY IONIZATION AND SECONDARY EFFECTS

In general, this type of noise is responsible for the difference between the thermal noise and the total noise under the desired condition that the filaments of the tubes operate at temperature saturation. A direct calculation or measurement of the effect of the several possible contributing agencies has never been made, and there will be attempted here only a short discussion of the more obvious contributing agencies.

The general theory of shot effect and of thermal noise takes account of all noise produced by the random emission of electrons from the filament and from their random movements in thermal agitation. Moreover, if the filament operates at temperature saturation, any kind of variation which may occur in the rate of emission from the filament produces no effect upon the space current. However, in actual operation, many electrons and positive ions are produced in the space charge region by collisions with gas molecules in the tube or by bombardment of the plate or grid.

Electrons produced in such a manner by ionization of the gas in the tube or by bombardment of the grid are drawn to the plate and produce noise in a manner similar to those electrons emitted by the filament when there is no space charge. It is therefore possible to form a rough estimate of the probable magnitude of the noise from the electrons from gas ionizations on the basis of plausible assumptions and the shot effect formula. Such an estimate indicates that in rather extreme cases, such as when the plate resistance is quite high and the gas pressure within the tube somewhat above normal, the noise from the ionization electrons may amount to a noticeablefraction of the thermal noise. Under ordinary conditions, however, noise from these electrons should be negligible.

When the positive ions which result from the ionization of gas or from bombardment of the electrodes are considered, the result is quite different. Instead of being drawn off by the plate, these are attracted into the space-charge region where small disturbances in equilibrium cause comparatively large changes in the space current. It is therefore to be expected that nearly all of the residual noise in vacuum tubes may be attributed to disturbances set up in the space-charge region by the entrance of positive ions liberated either from gas molecules, from bombardment of the elements, or by recombination of electrons in the space-charge region with ions evaporated from the filament itself.

Certain data to be discussed in the following section indicate that the plate is the most fruitful source of these disturbing ions, or at least that ions from the plate exert a more significant effect upon the noise than do those from filament or grid, but a much more searching investigation than was here attempted will be necessary before the exact role played by ions and secondary electrons in noise production may be determined.

Thus far, neither experiment nor theory has been successful in isolating the ionization and secondary effects enumerated above. It is therefore impossible to say that all of the noise sources have been included in this discussion. Nevertheless, the difference between the measured noise and the noise from thermal agitation and residual shot effect is of the probable order of magnitude of these effects, and no other sources of sufficient importance to warrant consideration have thus far been discovered. For the present, then, we turn our attention to the experimental results which show the comparative importance of the several effects, and demonstrate how they vary under different conditions of operation.

Part IV

EXPERIMENTAL RESULTS

In the introduction, noise sources discussed in the foregoing paragraphs were grouped under the three general headings:

1. Shot effect
2. Thermal agitation
3. Ions and secondary electrons

The scope of the first two groups is fairly well defined, and at high frequencies magnitudes may be estimated for the ideal case from the theoretical formulas given in the notes at the end of this paper. The third heading presents more difficulties, especially in the assignment of the relative importance of the several agencies which produce secondaries. The measurements to be described deal with noise produced on the plate side of the tube, since the noise produced on the grid side, in the absence of direct grid current, results entirely from thermal agitation, and has been discussed experimentally by Johnson.[1] In the experimental investigation of noise produced on the plate side, care was taken to keep the impedance between grid and ground so low that noise on the grid side might be neglected.

An amplifier with a voltage amplification of a millionfold was employed. This amplifier terminated in a vacuum-tube voltmeter, of the "negative C" type, which was operated only in the region where its response curve followed a square law to within ten per cent. The mean square noise voltage on the grid of the voltmeter was thus directly proportional to the deflection of the meter.

Two types of amplifiers were used. The first was a straight radio-frequency amplifier employing screen-grid tubes, with which measurements could be made over a range of frequencies from 700 to 1500 kc. The band width of this amplifier was about 4000 cycles. The other amplifier consisted of two stages of high-frequency amplification covering a frequency range from 2 to 20 megacycles, which was followed by a heterodyne oscillator-detector and a 300-kc, intermediate-frequency amplifier, with a band width of approximately 12,000 cycles. Screen-grid tubes were employed in both the high and intermediate-frequency stages.

Both amplifiers were carefully shielded against outside influences, and against "pickup" of the amplified noise from the latter stages, which, it was found, could become troublesome. The final measurements were made in a completely shielded room.

In order to avoid the necessity of determining the amplification of the amplifiers with each set of data the thermal noise from an anti-resonant circuit of known characteristics was used as a reference. Large error incident to a measurement of the large amplification at high frequencies was thus eliminated.

The two amplifiers were used to check the various noise sources until it was determined that nothing in the range covered, 700 kc to 20 megacycles, depended upon the frequency. Some preliminary work showed that at 300 kc the results were still in line with those at the higher frequencies.

It may be concluded that no source of noise not included in the theoretical discussion in Parts I, II, and III, and which depends upon frequency, is present in appreciable amount between 300 and 20,000 kc, although Johnson[4] has shown that at low frequencies, and especially below 1 kc, the frequency becomes effective in determining the noise. This he ascribes to time changes of the activity of the filament or to the behavior of the factor $\partial I/\partial J$ (see Note 1) at low frequencies. For radio frequencies these effects disappear with full temperature saturation of the filament, and hence above 10 kc the frequency at which the measurements are made is of no consequence when the full space-charge condition of temperature saturation holds. This statement would be expected to apply even in the region of very high frequencies above 40 megacycles discussed by Ballantine,[6] with the exception that secondaries from the grid may be expected to produce somewhat less noise when the time consumed in their passage to the plate is comparable with the time of a cycle of the frequency under measurement.

With the question of frequency disposed of, we may investigate the effect of the tube upon the noise by means of the arrangement shown in Fig. 4. In this connection the noise from the tube and the anti-resonant circuit is measured as the filament temperature of the tube is gradually raised.

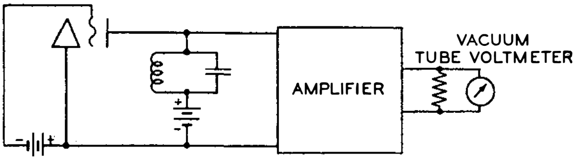

Fig. 4—Circuit for measuring noise on the plate side of a vacuum tube.

An example of the kind of result to be expected from such a procedure is given in Fig. 5-a, where the abscissas represent the power expended in heating the filament and the ordinates represent the resulting noise energies. When the filament is cold, the measured noise is that from the anti-resonant circuit alone. As the filament is heated so that electron emission commences, the noise curve mounts rapidly because of the shot effect. However, the space charge soon comes into play and causes the noise curve to bend over, while at the same time the lowered resistance of the vacuum tube contributes still further to the bending over of the noise curve.

The characteristic behavior of only the shot-effect component of the noise is shown in Fig. 5-b. The noise starts up along a curve proportional to the space current. Somewhere near the point A in the figure, the space charge begins to show its influence and the curve accordingly bends over toward the right. When the full space-charge condition has been reached, so that a change in filament emission does not change the space current, the pure shot-effect noise falls to zero, as shown at the point B in the figure.

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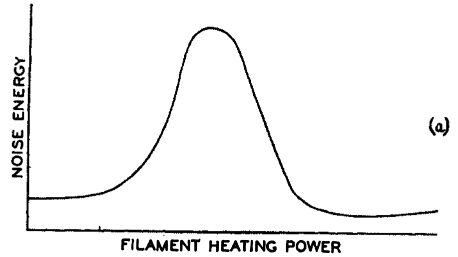
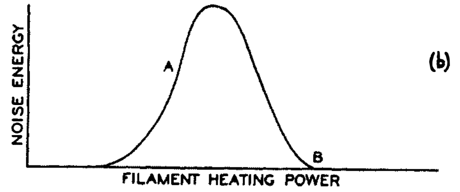
[6] "Schrot-effect in high frequency circuits," *Jour. Franklin Inst.*, **126**, No. 2, p. 159.

Meanwhile, there are two separate agencies operating on the part of the noise that comes from the thermal effect. First, the reduction in the plate resistance decreases the impedance between the plate and filament and so tends to reduce the noise, while secondly, the increase in the filament temperature produces a corresponding increase in the effective temperature of the plate resistance which tends to increase the noise.





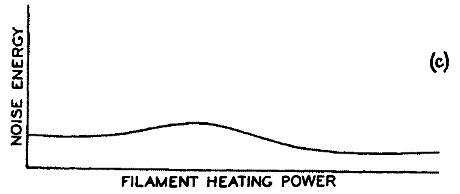


Fig. 5-a—Total noise energy from ideal tube with no ions and secondaries as a function of filament heating power.

Fig. 5-b—Noise energy from shot effect only as a function of filament heating power.

Fig. 5-c—Noise energy from thermal agitation only as a function of filament heating power.

The form of only the thermal noise curve is shown in Fig. 5-c. When the filament is cold the noise is taken as unity, so that the noise energy scale refers to the noise from the resistive component of the circuit impedance external to the vacuum tube. As the filament is progressively made hotter, the corresponding increase in temperature of the plate resistance at first produces an effect which preponderates over the decrease in the impedance produced by the decreasing value of the plate resistance. The noise curve accordingly rises somewhat above the unity value of the external circuit alone. The decreasing value of the plate resistance finally shows its influence by decreasing the noise, so that in tubes with low plate resistances the thermal noise with the filament hot may actually be less than with the filament cold.

Figs. 6 and 7 show these effects as calculated from experimental data. The dotted curve B represents the thermal noise as calculated by the method outlined in Part II and illustrated in Fig. 3. The dash line curve C shows the calculated shot-effect noise, while the solid line curve D is the sum of the calculated shot and thermal noises, and should therefore equal the measured noise in an ideal case where the effect of secondaries and ions is zero. Actually, the measured noise as shown by curve A fell below the calculated noise in the shot effect region. The reason for this must be looked for in the method of calculating the shot effect.

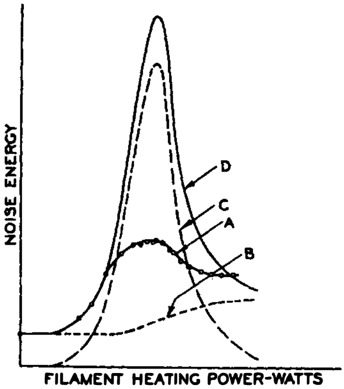


Fig. 6—Variation of noise with filament heating power for a three-electrode tube with tungsten filament.
Curve A shows the total measured noise.
Curve B shows the calculated thermal noise.
Curve C shows the calculated shot noise.
Curve $D=B+C$, shows the calculated thermal and shot noise.

Such a calculation involves a determination of the saturation current from the filament, and also of the rate of change of the actual space current with saturation current. A small error in the determination of the saturation current may therefore be expected to produce a very large error in the computed result, and this was found to be the case. The difficulties inherent in the measurement of the saturation current result primarily from two causes, although other things such as end cooling of the filament, occluded gases, and the potential drop along the filament also complicate the problem.

The first of the primary difficulties is the Schottky variation of saturation current with the applied electric field. The theory requires the evaluation of the saturation current at the operating plate potential. Therefore the attempt was made to find its value by the following procedure: The actual space current was plotted as a function of the filament heating current. When plotted on a power emission chart* the resulting curve approaches a straight line for very low values of heating power, but departs considerably for higher values of heating. The straight line portion of the curve was continued as a straight line up into the region of higher values of heating power. This straight line was taken as the saturation current. It may readily be realized that a small error in drawing the straight line may result in large errors in values for the saturation current.

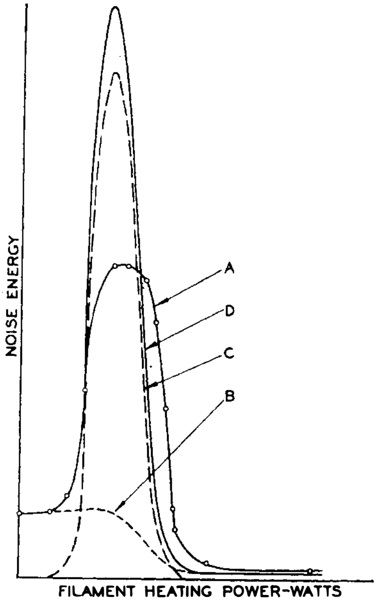


Fig. 7—Variation of noise with filament heating power for a two-element tube with oxide-coated filament.
Curve A shows the total measured noise.
Curve B shows the calculated thermal noise.
Curve C shows the calculated shot noise.
Curve $D=B+C$, shows the calculated shot and thermal noise.

The second of the primary difficulties in the determination of the saturation current results from the fact that immediately after a change has been made in the filament temperature, the saturation current continues to vary over a considerable length of time[7,8] when coated fila-

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* A power emission chart is a system of curvilinear coordinates designed by Dr. C. Davisson of these Laboratories. If the current obeys Richardson's equation and the cooling follows the Stefan-Boltzmann law, then the result when plotted on the chart gives a straight line.

[7] Davisson and Germer, "The thermionic work function of tungsten," *Phys. Rev.*, **20**, 300, 1922.

[8] Davisson and Germer, "The thermionic work function of oxide coated platinum," *Phys. Rev.*, **24**, December, 1924.

ments are used. When pure tungsten is employed, there is reason to believe that the same kind of an effect occurs, but in so short a time that ordinarily it cannot be separated from the time taken for the actual temperature change of the filament to become established. In the calculation of the shot effect, the "dynamic" value of $\partial I/\partial J$ is required. In view of the time variation mentioned above, no method of measuring this has been found. Accordingly the "static" values were used; that is, the final values reached by the currents after sufficient time had elapsed for steady state conditions to be reached.

When all of these computations were carried out, the result gave the curves shown in dashed lines on Figs. 6 and 7. It is seen that in both instances the results are too large. This was to be expected from the difference between the dynamic and static values of $\partial I/\partial J$. However, the important thing is that despite the complicated processes of measurement and estimate employed to arrive at the result, the curves are of the proper shape, and their maxima fall at very nearly the correct positions. Again, while the absolute values of the ordinates are too large, they are nevertheless of the same order of magnitude as the measured ones, and this fact lends support to the theory.

The curves of Fig. 6 refer to a tube having a tungsten filament and operated with 100 volts on the plate and -23 volts bias on the grid. The large grid bias was necessary for this tube in order to secure temperature saturation at normal filament temperatures.

Fig. 7 refers to a tube with an oxide-coated filament connected as a rectifier, with the plate and grid tied together. This was done to determine whether any new and fundamentally different effects were introduced by the different mode of operation. From several such curves as compared with those from the same and from different tubes with the ordinary three-electrode connection, it was ascertained that no unexpected results are attendant upon the mode of connection of the tube into the circuit.

Fig. 3 has been mentioned in Part II where it was pointed out that the solid lines represent the thermal noise calculated for the plate and its associated circuits. With filaments operating at temperature saturation, noise data on a number of tubes were taken. A few of these are representative of the entire number, and are shown connected by the dotted line curves in the figure. Curves *A*, *B*, and *B'* were for tubes with oxide-coated filaments operating at about 1000 deg. K. Curves *B* and *B'* represent the same tube, but in *B* the plate resistance was varied by means of the plate-battery potential, while in *B'* the grid-biasing battery was used to vary the plate resistance. It is seen that large values of negative grid bias produce a marked increase in the noise.

This evidence gives weight to the view that the difference between the actual noise and the thermal noise is mostly the result of positive ions which move into the space-charge region. The more negative the grid is made, the more pronounced is its effect in drawing the ions into the space-charge region.

Curve *C* shows data taken with a tube having a thoriated tungsten filament operating at approximately 2000 deg. K under the same experimental conditions that were used in obtaining curve *B*. As was to be expected, *C* lies higher than *B* by virtue of its higher temperature. Curve *A* was made for a tube with oxide-coated filament at 1000 deg. K. The amplification factor of this tube was 30 as compared with 6 for the tubes corresponding to curves *B* and *C*. For low values of plate potential curve *A* approaches the theoretical curve very closely.

A significant set of data from a practical standpoint is shown in Fig. 8. These data are the result of tests made upon a number of tubes

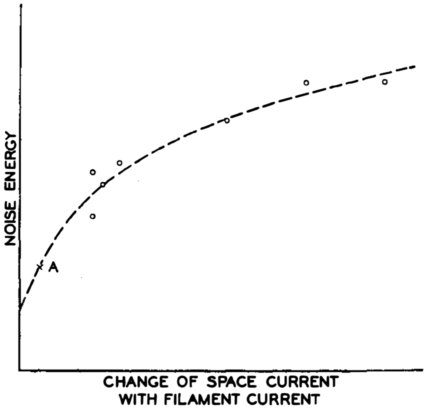

Fig. 8—Slope of a space current—filament heating power curve plotted against noise for a number of different tubes of the same type. Point *A* refers to a tube of another type having good filament temperature saturation.

all of the same type and show the effect of non-saturation of the filament upon the noise. The tubes were operated with the values of plate and grid potentials and filament temperature recommended by the designer. Under these conditions it was found that the filaments were in very imperfect temperature saturation. For the purpose of the experiment it was fortunate that the degree of temperature saturation was quite different for the different tubes so that a relation between the degree of saturation and amount of noise could be obtained. In this way, a rough though significant curve was secured which shows that the more complete the saturation the less was the noise. In the same figure, point *A* shows the same measurement made upon a tube of another type, having about the same values of plate resistance and amplification factor, but having quite perfect temperature saturation.

Tests for gas were made on a number of tubes, but no correlation between the gas pressure and the residual noise was found for any of the high-vacuum tubes in use today. Therefore, ionization by direct collision is thought to be of sufficiently rare occurrence to produce a negligible amount of noise.

A special study was made of the noise in the four-electrode type of tube commonly known as the screen-grid tube. In general, these tubes were found to have rather poor temperature saturation of the filament, and as a result showed a high noise level. The filaments saturated better and the noise level was much lower with 45 volts on the screen than with 67 volts. In rating these tubes for noise, it must be remembered that their high-plate impedance keeps the noise level quite high, so that an erroneous impression may be gained that they are too noisy for use where quietness is essential. Such is not the case. The signal is amplified to such an extent that the ratio of signal to noise ranks about the same as with other tubes having the same degree of filament saturation. There is room for a decided improvement in the degree of temperature saturation in all kinds of these tubes which have been tested, including those with the heater type of cathode.

Part V

GENERAL CONCLUSIONS

In the practical application of the foregoing theoretical and experimental study of the noise inherent in vacuum tubes and their associated circuits, there is one important fact to be borne in mind above all others. This fact is that the first stage of a radio receiving apparatus usually has a high impedance circuit attached to the grid of the tube. The thermal noise from such a circuit masks the noise from the plate side provided that the tube is operating as a detector or amplifier with temperature saturation of the filament. In this case, then, all that need concern the radio technician is the ratio of signal to noise in the input circuit.

When this is not the case the filament is nearly always responsible for the excess noise by exhibiting only a low degree of temperature saturation.

In order to be prepared to deal with those special cases or circuits where the input circuit impedance is sufficiently low to allow noise from the plate side to come into prominence, the more detailed investigation was undertaken. Here, again, it is found that with tubes properly designed to operate at temperature saturation, the major portion of the noise comes from thermal agitation in the plate circuit, with the internal plate resistance of the tube acting at the filament temperature. There is thus an advantage in tubes having saturation at low values of filament temperature.

The question of the shot effect in the presence of space charge has been dealt with theoretically. It is thought that the explanation here given may be subjected to a mathematical treatment suffiiciently rigorous to supplement the experimental tests, which are, for reasons explained, somewhat unsatisfactory. The practical criterion is theoretically predicted and experimentally established that perfect temperature saturation of the filament under operating conditions results in a reduction of the noise.

The exact effect of ions and secondary electrons upon the noise is not determined. It is certain that they contribute a measurable amount, but a more definite statement at the present time seems to be impossible just as an accurate determination of their effect upon the space current itself has never been made. It is found, however, that a large negative bias on the grid is harmful.

In conclusion the writer wishes to thank the many members of the technical staff of the Bell Telephone Laboratories who by their aid and suggestions have contributed to these noise studies.

Note 1

SHOT EFFECT IN THE PRESENCE OF SPACE CHARGE

The total space current of a given vacuum tube is dependent only upon the plate and grid potentials and upon the total electron emission from the filament, provided that thermal effects are neglected. Thus

$$I=I(E_p,\ E_g,\ J) \tag{1}$$

where

I is total space current
E_p is the plate potential
E_g is the grid potential
J is the total current emitted by the filament.

From (1)

$$\delta I=\frac{\partial I}{\partial E_p}\delta E_p+\frac{\partial I}{\partial E_g}\delta E_g+\frac{\partial I}{\partial J}\delta J\ \cdot \tag{2}$$

In this expression $\delta\ J$ may be interpreted as the change in the current emitted by the filament. Since no current ordinarily flows in the external grid circuit, δE_g may be taken as zero. Then, since by convention

$$\frac{\partial I}{\partial E_p}=\frac{1}{r_p}$$

equation (2) becomes

$$\delta I = \frac{\delta E_p}{r_p} + \frac{\partial I}{\partial J}\delta J \cdot \tag{3}$$

Imagine an impedance to be placed in the external plate circuit, and consider a frequency range between ω and $\omega + d\omega$. We may write

$$Z(\omega)\delta I(\omega) = -\delta E_p(\omega) \tag{4}$$

where Z (ω) is the impedance in the external plate circuit at frequencies between ω and $\omega + d\omega$.

From (3) and (4)

$$-\delta E_p(\omega) = \frac{\partial I}{\partial J} Z_0(\omega)\delta J(\omega) \tag{5}$$

where

$$\frac{1}{Z_0(\omega)} = \frac{1}{r_p} + \frac{1}{Z(\omega)} \tag{6}$$

so that $Z_0(\omega)$ is the impedance at frequencies between ω and $\omega + d\omega$ of the parallel combination of r_p and $Z(\omega)$. If an amplifier whose voltage step-up is given by the gain, $G(\omega)$, is arranged to amplify the voltage given by (5), then the voltage across the terminating impedance of the amplifier is

$$e(\omega) = \frac{\partial I}{\partial J} Z_0(\omega) G(\omega) \delta J(\omega) \cdot \tag{7}$$

But the right-hand side of this expression is merely the fraction $\partial I/\partial J$, *multiplied by the voltage that would be produced if there were no space charge.* The mean square value of this latter may be written, from Fry's formula:

$$\overline{V^2} = \frac{\epsilon J}{\pi} \int_0^\infty |\, Z_0(\omega)\,|^2 |\, G(\omega)\,|^2 d\omega \tag{8}$$

where ϵ is the electron charge, $= 1.59 \times 10^{-19}$ coulomb. Hence, the shot effect mean square voltage, with or without space charge, becomes

$$\overline{e^{2}} = \left(\frac{\partial I}{\partial J}\right)^2 \frac{\epsilon J}{\pi} \int_0^\infty |\, Z_0(\omega)\,|^2 |\, G(\omega)\,|^2 d\omega . \tag{9}$$

Note 2

EVALUATION OF $\partial I/\partial J$ FOR IDEAL CASE

In order to evaluate the fraction $\partial I/\partial J$, we note that it is the rate of change of the actual space current with the total current emitted by the filament, under the restriction that the values of plate and grid potentials are constant. Fig. 9 shows the manner in which I varies with filament temperature for various values of the plate potential. We may write

$$\frac{\partial I}{\partial J}=\left(\frac{\partial I}{\partial T}\right)\left(\frac{\partial T}{\partial J}\right).$$

From Fig. 9, the slope of the space current curve for a given value of plate potential, say 100 volts, is equal to $\partial I/\partial T$. Therefore, for the temperature T_1, the value of $\partial I/\partial T$ is given by the slope at P, as shown. Irrespective of the plate potential, the value of $\partial J/\partial T$ is given by the slope of the J curve at the point Q, corresponding to the temperature T_1.

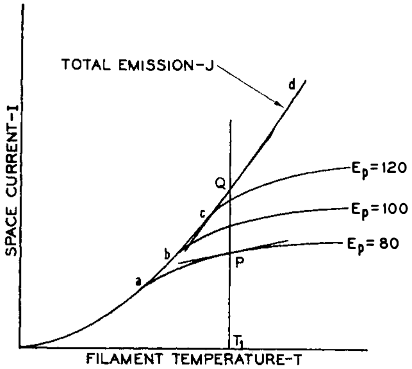

Fig. 9—Ideal variation of space current with filament temperature for various values of plate potential. The curve *Oabcd* represents J, the total current emitted by the filament.

For actual computation work it is convenient to plot the actual current I as a function of the saturation current J so that $\partial I/\partial J$ may be found directly.

Note 3

WHAT IS THE EFFECTIVE TEMPERATURE AS REGARDS PRODUCTION OF THERMAL NOISE OF THE INTERNAL PLATE RESISTANCE OF A VACUUM TUBE?

The difficulty in answering this question lies in the fact that the internal resistance of a vacuum tube is not a physical piece of apparatus but is a mathematical concept which measures the energy dissipated within the tube by a small variable current component through it. It is known from the kinetic theory of gases that the temperature of the electron cloud emitted by the filament is the same as the temperature of the filament itself. An analogous example is found in the case of boiling water, where the steam and water are at the same temperature. The electron cloud does not in itself, however, constitute the internal resistance of the tube. The electrons from the cloud are constantly being drawn off by the plate and resupplied by the filament. They acquire kinetic energy on their way to the plate. This energy does not affect the cloud temperature because it is coordinated energy, while the energy by which temperature is measured is the average random uncoordinated energy. Upon striking the plate the electrons dissipate energy. Here, then, is an attribute of resistance; namely the dissipation of energy when the electrons strike the plate. The coordinated kinetic energy which the electrons have acquired on their way to the plate is transformed into the random motions of heat energy of the material composing the plate.

Without, however, entering into a more detailed study of how the energy dissipation takes place, we may obtain the answer to our question by the following thermodynamic argument:

Refer to Fig. 10. This figure shows a vacuum tube composed of cathode, anode and grid, together with the d-c generators necessary to supply a positive potential to the plate and a negative potential to the grid. A resistance R is connected in the external plate circuit. The resistance R and the tube are placed in an oven where both are maintained at the same temperature T, which is taken to be high enough to insure a copious emission of electrons from the cathode. The cathode may be imagined to be oxide-coated, whereas the grid and anode are composed of a material which does not emit electrons. Moreover, the grid potential is adjusted so that a change in electron emission by the cathode does not change the space current. The absence of any shot effect is thus provided for, so that no energy other than that needed to maintain a perfectly steady space current is supplied by the generators.

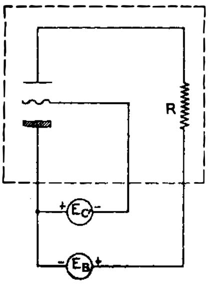

Fig. 10—Diagram to illustrate explanation of the effective temperature of the internal resistance of a vacuum tube.

Under these circumstances, the thermal agitation of electricity within the resistance R will cause varying currents to flow through the tube, supplying power to the internal resistance r_p of the tube. Therefore, the second law of thermodynamics requires that the internal resistance r_p must supply exactly the same power to the external resistance R. Suppose the two resistances to be equal in magnitude. Nyquist's formula tells us the mean square value of the thermal e.m.f. within R at the temperature T. Therefore, the mean square value of the e.m.f. within r_p must be exactly the same.

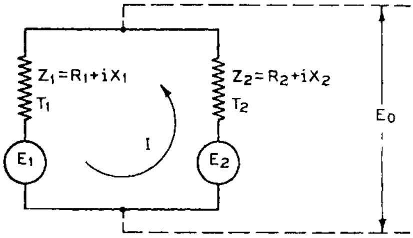

Fig. 11—Diagram to illustrate derivation of the expression for thermal noise from two impedances at different temperatures, T_1 and T_2.

But the temperature T is the cathode temperature. If the anode is cooled by circulating water, the energy relations between R and r_p are still undisturbed, since R is powerless to deliver electromagnetic energy to the cooling water.

The conclusion follows, therefore, that the effective temperature of the internal resistance of a vacuum tube is equal to the cathode temperature.

Note 4

THERMAL NOISE FROM TWO CIRCUIT ELEMENTS IN PARALLEL BUT AT DIFFERENT TEMPERATURES*

From the method of derivation of equation (1), the mean square value of the effective e.m.f. acting within an impedance of resistive component $R(\omega)$ and between the frequencies given by ω and $\omega+d\omega$ is

$$\overline{E^2}d\omega=\frac{2kT}{\pi}R(\omega)d\omega. \tag{1}$$

Consider the circuit shown in Fig. 10. The thermal electromotive forces acting in Z_1 and Z_2 are represented by E_1 and E_2, respectively, and the two impedances are at the different temperatures, T_1 and T_2. For the present, consideration is limited to those frequencies lying between ω and $\omega+d\omega$.

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* The modification of the formula for thermal noise which is here presented was derived by both Nyquist and Johnson independently of the writer, and at a previous time.

From the figure:

$$I=\frac{E_1+E_2}{Z_1+Z_2}.$$

Hence, the voltage E_0 is:

$$E_0=E_1-I_1Z_1$$

$$=E_1\frac{Z_2}{Z_1+Z_2}-E_2\frac{Z_1}{Z_1+Z_2}.$$

The mean square value of E_0 is therefore

$$\overline{E_0^2}=\overline{E_1^2}\left|\frac{Z_2}{Z_1+Z_2}\right|^2+\overline{E_2^2}\left|\frac{Z_1}{Z_1+Z_2}\right|^2 \tag{2}$$

From (1) the mean square values of E_1 and E_2 may be obtained. When these are substituted in (2) there results:

$$\overline{E_0^2}d\omega=\frac{2k}{\pi}\left(\frac{T_1R_1|Z_2|^2+T_2R_2|Z_1|^2}{|Z_1+Z_2|^2}\right)d\omega$$

or, by integration over all frequencies

$$\overline{E_0^2}=\frac{2k}{\pi}\int_0^\infty\left(\frac{T_1R_1|Z_2|^2+T_2R_2|Z_1|^2}{|Z_1+Z_2|^2}\right)d\omega \tag{3}$$

which may be put into the form used in the text by ordinary algebraic manipulation.

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