The Physics of Music, Part 5: Resonance

How strings and tubes produce tones

By Bill Markwick

If we leave electronic musical instruments aside for the moment, it's safe to say that all the music we listen to comes from something resonating, or emitting a tone at a specific frequency. Over the centuries, instrument makers seem to have tried every possible method of utilizing objects that can be made to emit desired tones. Some are simple and some are elaborate combinations of various resonator types.

In general, musical tones are emitted by stretched strings, strips or bars of various shapes (vibes, reeds, bells), pipes (whistles, organs, horns), diaphragms (drums) and a resonator that consists of an enclosed volume, the Helmholtz resonator (guitar bodies, etc). In addition, many instruments use combinations of the above.

Strings

The enormous string family includes guitars, the violin subfamily, banjos and a host of others. They may well have derived from the hunting bow, when somebody noticed that the twang could be changed in pitch by varying the tension. I'd also include the piano in our list; there's some dispute here, since it's actuated by hammers and might be called a percussion instrument, but there's no doubt that its tones come from stretched strings.

The textbook example usually consists of a string tensioned between two immovable posts. When plucked, it first forms a bow shape, like a skipping rope. This bow moves back and forth at a fixed rate; if the sound produced is picked up by a microphone and displayed, we see the familiar sine wave. The string can also generate overtones (called harmonics if they're musically related); the bow first splits in two, forming two equal bows. This produces a harmonic note (the second harmonic – the fundamental is the first harmonic in technical terminology) at twice the fundamental frequency. Now it splits into three, generating the third harmonic. The process continues, generating a whole series of notes at integer multiples of the fundamental frequency. Of course, all this happens in an instant, and all the harmonics sum together to form one complex tone.

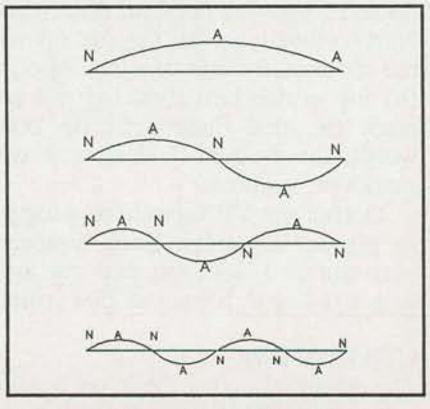
The points at which the string is at rest are called *nodes* and the maximum excursions called *antinodes*. Guitarists will be familiar with what happens when a string is plucked at a node point; the harmonics with that node are accentuated. If there's a guitar handy, strum the strings over the twelfth fret and compare it to plucking near the bridge. This is the midpoint of the string, and the even harmonics will be accentuated, producing a soft, mellow sound.

If you'd like to hear the second harmonic by itself (almost), put your finger lightly on any string over the twelfth fret and pluck the string sharply near the bridge. You'll hear the high-pitched, singing second harmonic quite loudly, mostly because we've damped out the fundamental with your finger over the fret. Other harmonics can be produced at other points; two octaves by touching the string over the fifth fret (one quarter of the string length), an octave-plus-fifth (third harmonic) over the seventh (which divides the string in the ratio of 3:2) and so on.

If you'd like to hear the fundamental pretty much by itself, touch the string firmly about half a finger's width from the bridge and pluck it sharply. It's a fairly loud but very dull sound because you've damped most of the harmonics.

Wooden Amplifiers

The classic case of the string stretched between two immovable points doesn't occur in musical instruments. If it did,



The nodes and antinodes of a vibrating string for four harmonics.

you wouldn't hear much, because the string's small surface area won't set enough air in motion. It's necessary to direct the energy of the string via its termination (the bridge) into a panel which will match this energy to the surrounding air. Usually the panel is a thin wooden soundboard, as in the violin family, guitars, pianos, etc. To further enhance the tone, certain harmonics of the string/top combination can be controlled by mounting the panel on top of an enclosure. This enclosure, such as the violin or guitar body, has complex resonances itself, favoring the high frequencies in the stiffer areas and the low frequencies in the large, flexible areas. In addition, the air in the cavity has a resonance of its own (the Helmholtz resonance). The entire instrument forms an incredibly complicated sound generator.

Further, the resonating body feeds energy back into the string, greatly affecting the tone. This is why a solid-body electric guitar sounds quite different from an acoustic guitar with electric pickups; the solid body does not return as much energy to the strings.

The fundamental frequency at which the string vibrates varies inversely with its length, and also with the square root of its mass and the tension. The length is determined by the construction of the instrument; the tension and mass are more easily varied then the length to keep the size manageable. For instance, if you scale up the size of a violin exactly so that the pitch is lowered to that of an upright acoustic bass, the body alone would be about six feet high. To keep it playable, thicker strings under less tension are used.

The pitch of any given string is inversely proportional to its length; half the length produces the octave, and so on. It gets a little more complex with instruments that have fingerboards: when you press the string down, the tension is increased and the pitch goes up. The fret positions (or the violinist's fingering) must be adjusted to compensate for this. If you can accurately measure a guitar fingerboard, you'll find that the twelfth (octave) fret is not exactly half of the string's length. The banjo has an easily movable bridge, and banjo players will be familiar with the technique for replacing a removed bridge: its location is adjusted until the harmonic sounded over the twelfth fret is equal in pitch to the fretted note at the twelfth.

Windings

Musicians occasionally ask why the thicker strings of an instrument are wound with wire instead of being solid. The answer is that although the pitch depends only on tension, length and mass, the quality of the sound depends on many other factors, in particular the stiffness of the material. Short sections of a thick, solid string would be reluctant to vibrate, and this would attenuate the higher harmonics. Windings improve the flexibility and the brightness. Of course, they collect skin oils and dust, too, so the wound strings tend to go dull more rapidly than the solid upper strings. You can't have everything.

And can a string go out of tune with itself? The answer is yes. The ideal string's harmonics follow an integer rule: 1,2,3,4 times the fundamental and so on. If the string is dirty or starts to unwind, sections of it can vibrate at frequencies unrelated to the fundamental. In bad cases you can actually hear a sour tone or two floating above the fundamental, or the sound may be of indeterminate pitch. Lazy guitarists like myself who hate changing strings hear this effect all the time.

Bars

The vibrating bar is used in vibraphones, harmonicas, reed woodwinds and various percussion instruments, such as chimes. The bar is either clamped at one end (reeds) or free at both ends (chimes, etc). The bar's resonance depends entirely on its stiffness, unlike the string, which uses tension as the restoring force. The fundamental frequency is inversely proportional to the square of the length, and also depends on thickness, density and the type of material.

In the case of the bar clamped at one end, the overtones are not simple harmonics, but are related to the fundamental in ratios like 6.2:1 and 17.5:1. These non-integer relationships account for the "edge" on the sound of reed instruments.

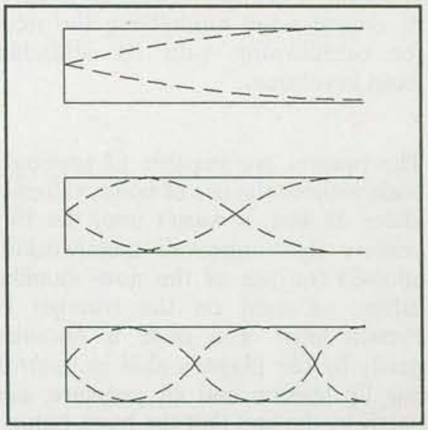
The bar free at both ends, as in vibes, xylophones, chimes, etc., has a somewhat different overtone structure, with ratios like 2.7:1, 5.4:1 and so on. These are still non-integer, but a little closer to musical harmonics, producing a mellow but rich bell- like sound. Pipes of certain lengths are often suspended under the bars to further accent the sound.

Pipes

The pipe open at both ends is a simple resonator: its resonant frequency is $c/2L$, where c is the speed of sound in air and L is the length. The overtones produced are integer harmonics: 1,2,3,4 times the fundamental and so on.

The pipe closed at one end has a frequency of $c/4L$ and its harmonics are odd integer multiples: 1,3,5, etc.

In both cases, the exact fundamental frequency depends on the shape of the end of the pipe: a flared pipe will have a slightly different effective length than a straight pipe. Since the correction is a fraction of the radius and is added to the physical length to get the acoustic length, it's often ignored for long pipes with a small radius.



The fundamental of a tube closed at one end is at one-half the wavelength. The first three harmonics show that an antinode is always at the closed end.

The resonance of the pipe gives us the pipe organ, woodwind instruments, brass instruments, and whistles; pipes are also used for special effects in other instruments (as in the vibraphone).

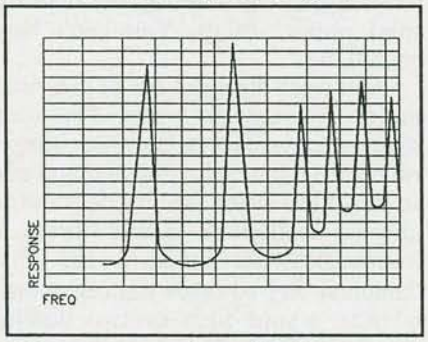
The whistle family and the pipe organ depend on the airflow over the pipe's aperture itself to make the sound (win a trivia contest: the sound-producing aperture of a pipe-style instrument is called a *fipple*); in brasses and woodwinds, the sound is produced by a reed (the lips function as a reed in the horn family). The pipe organ has a pipe for each note; other instruments depend on various methods of altering the pitch and producing a scale.

In the woodwinds and whistles, holes are covered or uncovered by fingertips or mechanical valves, effectively changing the length of the pipe. In the case of the inexpensive tin whistle (pennywhistle), over two octaves is possible from the six holes. If the holes are covered and the force of blowing increased, the pipe will jump to the octave note, twice the fundamental frequency. Now the player can start over and play the scale in the second octave. It's also possible to overblow even harder and jump to the next higher harmonic, which is the fifth above the octave (that is, on a C whistle the harmonic would be G plus an octave). On instruments like the recorder, a thumbhole is drilled into the back of the cylinder. Opening this hole encourages the instrument to jump to its octave note, minimizing the need for overblowing with its attendant jump in volume.

The brasses are capable of playing a scale without the use of holes, valves or slides. In fact, it wasn't until the 19th century that improved metalworking allowed the use of the now- familiar valves, as used on the trumpet or French horn. The scale is obtained partly by the player's skill in controlling lip tension and air pressure, and partly by the fact that the horn, being a pipe, will naturally play a harmonic series. This series in the key of C goes: C-C-G-C-E-G and on up; these notes correspond to the integer harmonics, 1,2,3,4 times the fundamental and so on. The notes become closer and closer together and eventually an entire scale is generated. It's not exactly a perfect scale, with many of the notes straying from the pitch of the same note played on a piano or fretted instrument; we'll be investigating why in a future issue.

The natural, or valveless, horn can only play in one key. To get around this in past centuries, players could insert a crook (a length of extra tubing) to change the fundamental, or insert a hand in the bell of the horn to get a sharp or flat. Still, the larger part of its lower range is rather gap-toothed; we can only marvel at the immense skill of horn players in the past, or today's players who use original instruments. Once smoothly-operating valves and slides were available, the brasses became chromatic instruments capable of playing any note. The valves add extra lengths of tubing; the extra length can be used to lower the pitch of any note. For instance, if you wanted an F note from a C trumpet, you'd vary lip tension to produce the harmonic G, but with a valve down that drops the pitch a whole tone. It sounds complicated (and beginning trumpeters will agree that it is).

It's of interest to note that horn players have complained that modern reproductions of early instruments are hard to play and keep in tune compared to museum-piece originals. The reason turned out to be technology: the older horns were individually hand-made, with the accompanying small errors and variations in diameters and symmetry. This asymmetry gave the older horns fairly wide peaks at the harmonic frequencies, rather like a low-Q bandpass filter, making it easy for the player to "bend" the pitch to suit. Modern machine-made horns, by contrast, are symmetrical throughout their construction, resulting in a high-Q bandpass (a very tight peak at the harmonic frequencies). It's more difficult to play smoothly on the latter instruments.



The response of a typical horn. The resonant peaks correspond to the notes C-C-G-C-E-G.

The Helmholtz Resonator

Sounding a note by blowing over the neck of a pop bottle is a demonstration of a Helmholtz resonator. It's defined as an enclosed volume of air coupled to the outside air by means of an aperture. It produces the sound from police whistles or ocarinas, and enriches the sound of other resonators, as in guitars, violins, etc. The formula for its resonance frequency is rather involved, but essentially it depends on internal volume and the radius of the orifice.

The bodies of the violins, guitar, or any instrument made of wood are complex resonators indeed, with different sections responding to different frequencies. Guitarists who have tried installing a pickup in an acoustic guitar will have discovered this; the soundboard around the bridge favors low frequencies, while the soundboard closer to the neck radiates the higher tones. When this plate resonance is added to the air (Helmholtz) resonance of the internal volume, a fine guitar or violin becomes much more complicated than first appears. In fact, physicists have been studying the violin in depth ever since its development in the 16th and 17th centuries, and some aspects of it still remain a mystery, probably because there are so many variables in construction and playing techniques.

Stretched Diaphragms

The stretched diaphragm is used in drums, cymbals, the tambourine and the banjo. They are all circular, very thin and supported either at the circumference or in the centre (or in the case of the gong, free-standing). Since they produce very complex overtones, their sound is not always perceived as having a definite pitch, though the boom of the kettle drum can be tuned to an easily recognizable note. The fundamental frequency is found in much the same way as the stretched string: it varies with mass, tension and the radius of the diaphragm. The many overtones are not harmonically related, but sum to produce an instantaneous sound that closer to "controlled noise". The tension of the drumhead or banjo head is adjusted until the predominant harmonics suit the player and the instruments. In guitar construction, the maker will often adjust the thickness of the top until it seems to agree nicely with an A note.

In the next issue, we'll look at the arithmetic of the musical scale. We'll compare the equal-tempered scale and the natural scale, with some explanations as to why horn players, violinists and singers sometimes seem to be at odds with the notes from pianos and fretted instruments. ■