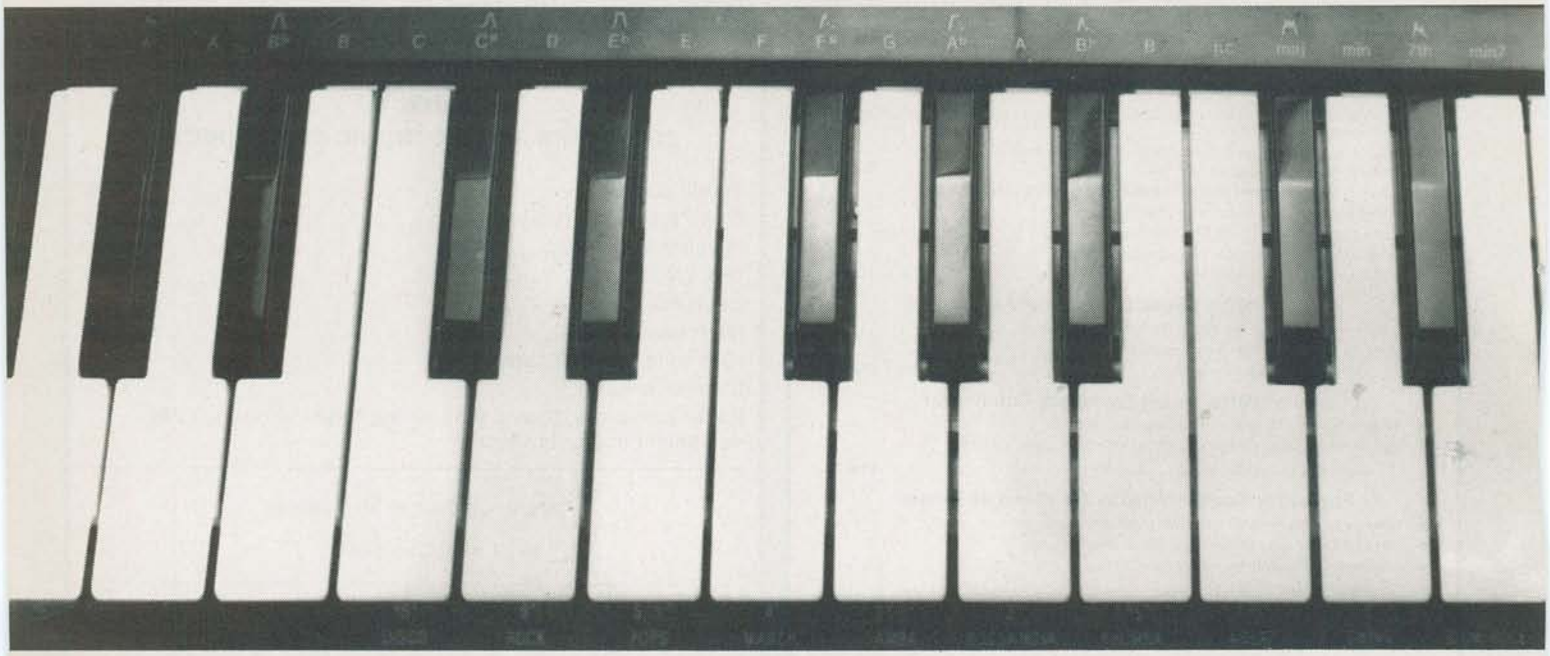
The Physics of Music, Part 6

***The natural scale, the equal-tempered scale and where they come from.* By Bill Markwick**



The notes of the musical scale, the do-re-mis planted in our heads from childhood, are so familiar to us that we take it for granted that these building blocks of music are just part of nature, a given, like *pi* or Newton's laws.

Not so. The choice of pitches for the note series could be based on anything: the year of your birth, your favorite number, you name it. The resulting music would sound weird to Western ears, or anyone's ears, but you could do it, and if it caught on for a generation or so, people would assume that *that* was the way music has to be.

However, we pointed out in past articles that any musical instrument produces harmonics, extra notes that combine with the fundamental to give the instrument its characteristic timbre (or flavor, if you like). These harmonics are usually integer multiples of the fundamental: twice the frequency, three times, four times, and so on up to inaudibility. The harmonics occur simultaneously to give the sound its character.

Suppose that you decided to have eleven notes in your scale, and that each ascending note was going to be 10% higher than the one below because you liked nice round numbers. If you had never heard any music before, you'd be as pleased with your scale as the person who invented the wheel (hey, the wheel is simple. How about the person who invented the *axle?*).

The trouble is, your strings and tubes and horns and whatnot are going to continue with their integer harmonics: the octave will be twice the fundamental, the next will be three times (or half again as high as the octave harmonic – a musical fifth). Horns, for instance, will still continue to play a scale based on the harmonics. To further plague your new invention, humans have the odd ability to detect when something is half or twice something else. People who have trouble tuning their guitars or violins can lock right onto the octave or fifth.

So when your music starts up, there'll be quite a mixture of your new notes and the harmonics determined by the physics of resonators. Since you've decided on an 11-note scale, a fundamental note of 100Hz will have an octave of 259.4Hz. When you play this octave for your listeners, they'll say "No, it isn't", because their brains want them to hear 200Hz for the octave. Most of your instruments will sound out of tune with themselves, as the integer harmonics grate up against the fundamentals that you've chosen.

This is not to say that your scale is wrong, not by any means. Various societies have concocted various scales throughout history, though I imagine most of them recognized the existence of the octave and fifth right away. The point is that if you want to be an original scale designer using a new system of temperament, you're bucking a very strong headwind. Composers who have come up with new scales, such as the twelve-tone, are really just rearranging the standard western chromatic scale; after all, their music has to fit on existing instruments.

Concert Pitch

Before we get into any number-crunching, it's best to clear up a misunderstanding that happens whenever anyone writes about musical frequencies. If you look in physics books and articles like this one, you'll find certain frequencies given for certain notes. If you look in music books, you'll find a different set of values for the same notes.

The reason for this is the science writer's wish that you understand the numbers without having to plug everything into a calculator. If we take a note as 100Hz, as we did above, the harmonic series is instantly apparent: 100, 200, 300, 400 and so on. The 100Hz note, however, doesn't exist on the piano.

In 1939, rather late in the history of music, there was international agreement to standardize the frequencies of the musical scale, and 440Hz for A above middle C was chosen as the reference point. We'll shortly come to the explanation of how to derive the rest of the notes.

When you see scales explained in terms of middle C being 256Hz, which it isn't, indulge the author of the work;

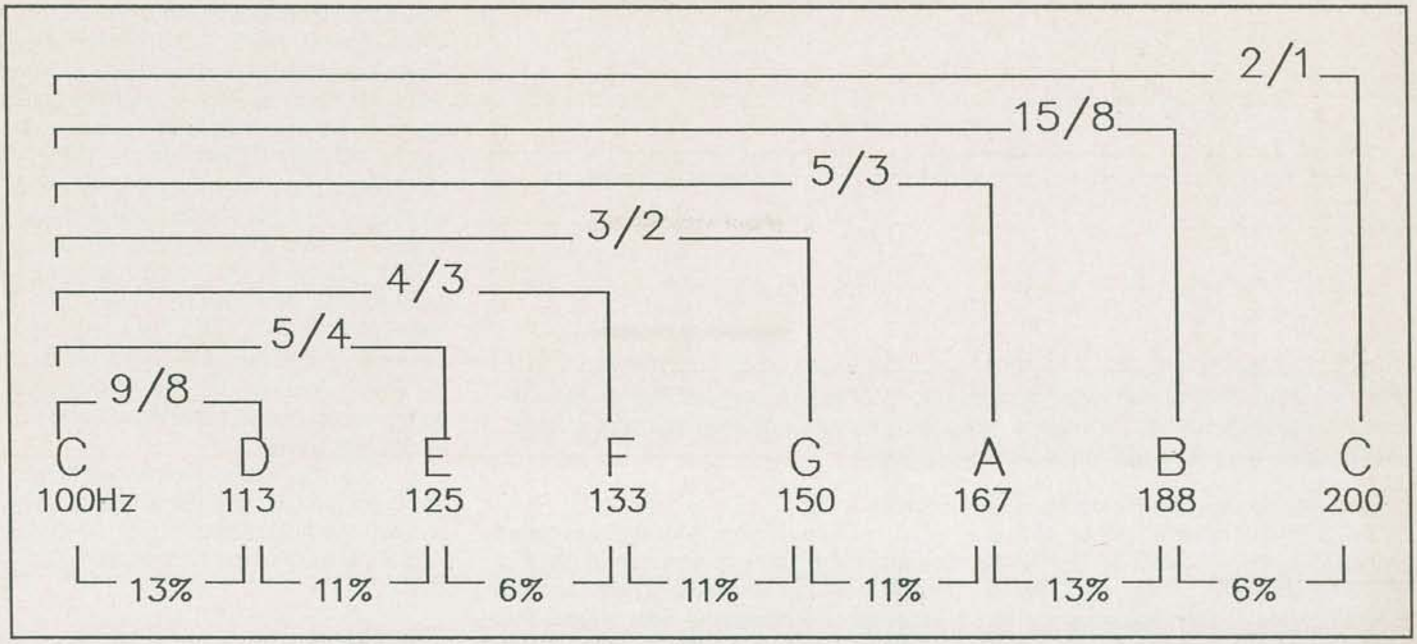


Fig. 1. The natural harmonics derive from simple integer ratios as shown above the scale. C is taken as 100Hz for clarity, and the frequencies of the other notes shown below them. The last row shows the percentage change from one note to the next; there are three types of intervals in the natural scale. The numerical values are rounded off to integers.

he or she is saving you from having to look at C being 261.63Hz, which it is.

Incidentally, it's of interest to note that technology has raised the pitch of music over the centuries. The early pianoforte, for instance, was limited in the amount of tension the older strings could stand, and the same probably goes for other stringed instruments as well. The A above middle C was about a semitone lower than 440Hz, about 415 to 425Hz according to tuning forks from the late 18th century.

Early Scales

Once you discover the fundamental, octave and fifth notes (1,2, and 3 times the fundamental), usually by experimenting with a simple whistle or horn, you might decide to expand your scale a bit by seeing what other notes you can make, and which of these sound good with the others. The next few notes in the harmonic series will be another octave, the third above this, and the fifth above this, corresponding to 4,5 and 6 times the fundamental. In terms of today's notation, here's what we have so far if we take the fundamental as C:

C-C-G-C-E-G

This note sequence will be familiar to horn players; it's the note sequence a horn will play without valves or slides. Continuing on with the process, primitive musicians would have found the second (D above C) and the fourth (F above C). Now we've really got something here: five notes, six if you count the octave: C-D-E-F-G-C. The problem is that they span a very large frequency range, about three octaves worth of very forceful horn or whistle playing, not to mention the gaps between some of the notes; a closely-spaced scale doesn't appear until rather high in the harmonic series where it's hard to play unless you have lungs of steel. To make the notes easier to get, folk musicians (as opposed to today's Folk Musicians, a different thing entirely) would have brought their newfound notes into the space of an octave by means of holes drilled in the whistle, or separate strings mounted on a frame, as with the harp.

Modes

These simple, gap-toothed scales are called *modes*, and a great deal of music has been obtained from them in many different cultures throughout history, and they're still in use today. If you have a harp that's strung C-D-E-F-G-C, you can play all sorts of simple tunes. As a trivial example, you can play *Mary Had a Little Lamb* with only C-D-E-G. There's no reason that you have to have C as the root note; a harp could be tuned D-E-F-G-C- D; this rearranges the tone\halftone sequence and produces a haunting sound somewhere between our major and minor scales. Different modes are easily obtained by starting on different keynotes. To illustrate this, trying playing on the piano from A to A, using only the white keys; this is the Aeolian mode, better known as our modern minor scale.

Naturally, musicians began to experiment with changing and expanding the five notes. By changing the note that you take as the root, they noticed that different-sounding "scales" were produced; no doubt they became dissatisfied with the large gaps between some of the notes, and added some more. The note A is from the harmonic series and fits nicely; in the key of C, this gives us a sequence close to the major scale with a gap just before the octave C. This gap was filled in one

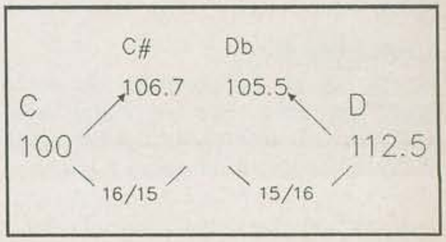


Fig. 2. The natural scale has both a sharp and a flat between the whole notes, as explained in the text.

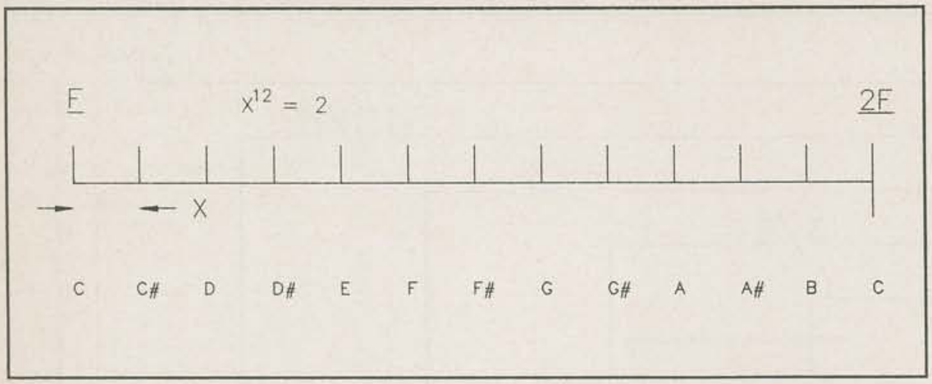


Fig. 3. The division of the octave into 12 equal sections produces the equal-tempered scale. X is a multiplier that will give us an octave frequency of twice the fundamental when multiplied by itself 12 times.

of two ways; musicians sometimes inserted a small interval above the A, giving the equivalent of C-D-E-F-G-A-Bb-C (Mixolydian mode, or flatted-seventh scale), and sometimes they added a whole tone, giving a major scale.

With an instrument tuned to an eight-note major scale, you can get all sorts of effects. If you have a keyboard handy, try taking different notes as the root, but stay on the white keys. If you play from A to A, for instance, you have our modern minor scale. G to G will produce the Mixolydian mode mentioned above. *Old Joe Clark* is a classic example of this mode; play it in G on the white keys only, starting with a D note. There are many other modes that can be constructed on the white keys, and they were all used to flavor the music with sharps and flats back when there *weren't* any sharps and flats; you slide the intervals around by changing your selection of root note.

Modal music tends to confuse a bit these days because the sound isn't quite major or minor, but seems to have characteristics of both. *The Wreck of the Edmund Fitzgerald* is an excellent example of modern songwriting in the Dorian mode: play it on the white keys from D to D, with the first note being A.

Changing Keys

Here's where we come to the main stumbling block with our major scale that we've created out of the harmonic series. Changing keys means to raise or lower the pitch while keeping the note relationships the same. Singers prefer different keys, and instrumentalists would have wanted more keys to avoid having to carry many single-key instruments.

Even if you add the necessary sharps and flats to our harp or flute, it isn't possible to switch to another key without encountering sour notes. The reason for this should be clear if you ponder the intervals shown in **Fig. 1.**

With our familiar equal-tempered scale, we're used to building music out of the tone and the semitone (half a tone). The tone increase from C to D, for instance, is the same percentage increase in pitch as from G to A; a tone is a tone. Further, sharping a note produces exactly the same pitch as flatting the note above; D# is the same as Eb and so on. Not so in the natural scale generated according to the laws of acoustics. Note that there are a number of unique percentage changes (the frequencies and percentages are rounded off to integers). Instead of the tone and halftone, we now have the major tone (about a 13% increase), the minor tone (about an 11% increase) and the semitone (about 6%).

The intervals of the natural scale are Tone, Minor Tone, Semitone, Tone, Minor Tone, Tone, Semitone. Even if you add sharps and flats, changing keys is going to mess up the order; imagine that you have a C# and F# available, and you try to play in D simply by playing a major scale starting on the D note. You're into sour trouble right away. The first interval, D to E, is a minor tone when it's supposed to be a major.

And if that's not enough confusion, consider that sharps and flats are completely different creatures in the natural scale. A semitone is defined by the harmonic series as an increase of 16/15, or a decrease of 15/16. Taking C as 100Hz, C# becomes 106.7Hz. Coming down from the D note of 122.5Hz, Db becomes 105.5. A keyboard tuned to the natural scale would have to have keys for both the sharps and the flats. In fact, an organ builder in past centuries actually made such a thing. I don't suppose it was very popular with organists. I've also heard that a guitar-maker in the US is bringing out a guitar with an 18 or 19 fret neck to accommodate the natural scale, but I haven't been able to find out any more about it. No doubt it's a fun exercise for the builder, but the improvement in temperament must be rather subtle for such complication.

So the natural scale gives the sweetest sound, agreeing as it does with all the harmonics ringing from the various notes. It just isn't very practical.

The Cure

Long before Bach did such a marvelous public relations job for equal temperament with his *Well-Tempered Clavier*, people had been experimenting with dividing the scale into 12 equal sections (along with the mean-tone system, which was a compromise that allowed a few keys to work well and the others badly). Each section is a semitone and each tone is two semitones. Since there is no bother with the major-minor-semitone sequence, the

C 100
D 112.2
E...... 125.9
F...... 133.5
G 149.8
A...... 168.2
B...... 188.7
C 200

C..... 261.6
D..... 293.7
E...... 329.6
F 349.2
G..... 391.9
A....... 440
B...... 493.9
C 523.3

Fig. 4. The upper column shows the frequencies of the equal-tempered scale for C = 100Hz, which is not a musical pitch and was chosen for convenience. The lower chart shows concert pitch for A = 440Hz.

root note can be anywhere; every scale in every key is identical to all the others except for pitch.

Fig. 3 shows how the intervals are derived. F is the frequency of any note, and 2F is the octave frequency above it. To find the frequency of the next semitone, F has to be multiplied by a number X that will give 1/12 of the octave. If you multiply our semitone by X, you should get the next note, and so on until the last note is twice F.

Looking at it another way, X times itself 12 times has to equal 2.

$$X^{12} = 2$$

Solving for X by taking the 12th root of both sides, X becomes the 12th root of 2. Poke this into your calculator or computer (easiest way: 2 to the 1/12 power) and you'll get 1.059463094.

If you multiply any note by this, you get the next semitone up. Multiply any note by the reciprocal (.943874313) and you get the semitone below.

Multiply it by 100 and you get the percentage change in frequency between notes: 105.946%, if we round it a bit. Thus any note is 5.946% higher than the semitone below.

The Catch

There's always a price to be paid in compromises, but in the case of the equal-tempered scale, it isn't much. In fact, as complex compromises go, it's a miracle.

Musicians with *very* good senses of pitch will argue that the notes are never exactly on, especially the pesky seventh or leading tone. Others complain that the natural harmonics from the instruments clash with the equal-tempered fundamentals. You can hear this last effect on a very well-tuned piano: sound a C and the G in the next octave above it and listen as the notes fade away. You should hear rapid beats weaving in and out of the sound. They're caused by the tiny difference between the natural harmonic G (off the C string) and the equal-tempered G string.

Still, all the differences are extremely small, small enough that you rarely hear a complaint about this. People who play continuous-tone instruments (violin, trombone) will often bend the notes to suit themselves anyway. I'm not much of a violin player, but on occasions when I'm having a good saw at it, I like the leading tone very close to the octave, as do a number of other players. Perhaps we've invented our own musical scale.

Fig. 4 gives you two frequency charts. The first is the equal-tempered scale of C with C taken as 100Hz because it made the arithmetic easier. The second is the proper equal-tempered scale, in real concert pitch, starting on middle C; I derived it by taking A as 440Hz and using the 1.05946 method outlined above.

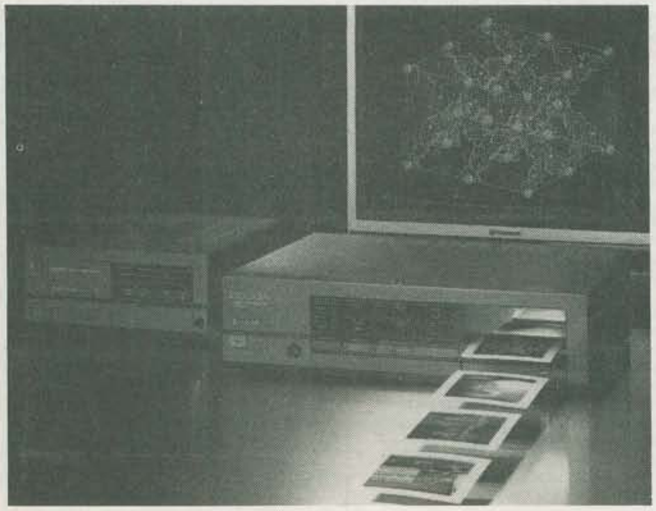
The Cent

To simplify working with small pitch changes, the octave is further divided into 1200 *cents*, with each semitone being equal to 100 cents. A semitone should really be called a dollar, then, and while we're at it, we need a name for 1.05946, like *fleen*.

The cent rarely turns up in music except on the readouts of electronic tuners.

Continued from page 8.

For Your Information



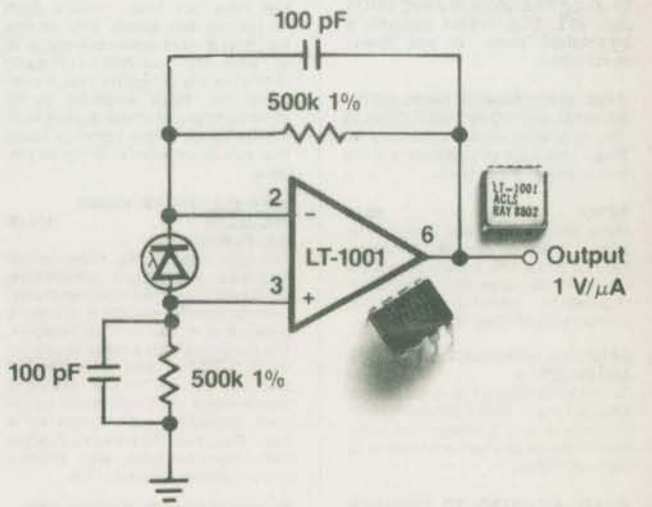
Video Floppy System

The new Hitachi Video Floppy System produces high resolution prints in full color of anything that can be viewed on the screen of a TV, freezing motion, recording events for future reference.

When the scene you want to keep is displayed on the TV monitor, press the memory button of the VY-50A or VY-100A Printer; the video information will be stored in IC memory to be printed out in 80 seconds after issuing the print command. The VX-52A floppy system is capable of storing up to 50 scenes on a standard 2" video floppy disc.

If you'd like more information on the Hitachi Video Floppy System contact: Rayonics Scientific Inc., 585 Canarctic Drive, Downsview, Ontario M3J 2P9. Ph: (416) 736-1600.

Circle No. 77 on Reader Service Card



New Op Amp

The LT1001 from Raytheon boasts an ultra low offset voltage of 15uV maximum, and low offset drift of 0.6uV/°C maximum. This makes the device ideal for low level signal conditioning, instrumentation, data conversion applications eliminating the need for offset zero adjust in many instances.

Additionally, the 1001 features a low input bias current of 2nA maximum, high CMRR of 114dB and PSRR of 110dB. Low input noise voltage is a mere 0.6uV p-p (0.1 to 10 Hz) and its power dissipation of 75mW.

More information can be obtained from Lorraine Jenkins, Raytheon Company, Semiconductor Division, 350 Ellis Street, Mountain View, CA 94043.

Circle No. 78 on Reader Service Card

Continued on page 28.