a dB Primer

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How to work with decibels and convert them to their electrical equivalent in various areas of electronics—from communications to hi-fi

THE expression of voltage, current, and power ratios in decibels (dBs) is pervasive in literature about, and analysis of, electronic circuits. Therefore, anyone interested in electronics, from audio through amateur radio, should clearly understand the concept of decibels. Here are decibel basics, using a minimum of math.

Gain and Loss. The amount of output power from a linear electronic network is proportional to the amount of power present at its input. Thus, the power lost or gained in such a network is proportional to the amount of input power, as shown in Fig. 1. When 10 watts of power are applied to the network's input (Fig. 1A), 9 watts are dissipated as heat and 1 watt appears at the output. When 1000 watts are applied to the same network (Fig 1B), assuming that it can safely handle this increased power level, 900 watts of heat will be produced with only 100 watts of output power.

The amount of power at the output of the attenuator, P_O, is related to the input power P_I by the equation $P_O = (K)(P_I)$ with $K = 1/10$; where K is a ratio called the gain factor. Of course, it is possible to cascade two or more such networks to obtain a cumulative effect, as shown in Fig. 2. Here two attenuating networks are used. Their total effect is identical to that produced by a single attenuator with a gain factor of 1/100: $P_O = (1/10)(1/10)(P_I) = (1/100)(P_I)$.

Cascading linear electronic networks results in the multiplication of their gain factors. It might be well at this point to mention that loss is treated as a fractional gain. For example, the 10:1 attenuators of Fig. 1 have gain factors of 0.1. Contrast those with the gain factors of most amplifiers, which are often appreciably greater than unity.

Defining the dB. It would be very convenient if we could express gain factors in such a way that they are additive in nature. Then the cumulative effect of cascaded gain or loss blocks could be calculated simply by adding terms, not multiplying them. The decibel allows us to do exactly that.

A decibel expresses a ratio—specifically, 1.259:1—so the addition of decibel gains is equivalent to the multiplication of ratios or gain factors. Power gain in decibels is formally defined as: $G(\text{dB}) = 10 \log_{10}(P_O/P_I) = 10 \log_{10}(K)$. Note that the logarithm of a positive number less than one is negative. Thus, negative decibels represent fractional gain or attenuation. Positive decibels signify gains greater than one or amplification. Applying the power formula to the attenuators of Fig. 1, we see that $G = 10 \log_{10}(1/10) = 10(-1)$ or -10 dB. Table I summarizes common power ratios and their gains in decibels.

Another Definition—dBm. Power levels are also expressed in dBm, that is, the number of decibels greater or less than a reference level of one milliwatt. Mathematically, this is defined as: $P(\text{dBm}) = 10 \log_{10}(P_{mw})$ or $30 + 10 \log_{10}(P_W)$, where P_{mw} is the power in milliwatts and P_W is the power in watts.

For example, 10 watts is 10,000 milliwatts, so $P(\text{dBm}) = 10 \log_{10}(10{,}000) = 10(4)$ or $+40$ dBm. Also, $P(\text{dBm}) = 30 + 10 \log_{10}(10) = 30 + 10$ or $+40$ dBm. One microwatt is 0.001 milliwatt, so $P(\text{dBm}) = 10 \log_{10}(0.001) = 10(-3)$ or -30 dBm. Table II lists common values of power in watts and milliwatts, and their counterparts in dBm.

Converting Back. One rarely needs to convert dBm or dB back into watts or power ratios; but for the sake of completeness, we will include the relevant formulas. To convert dBw into watts, milliwatts, or gain factors (power ratios),

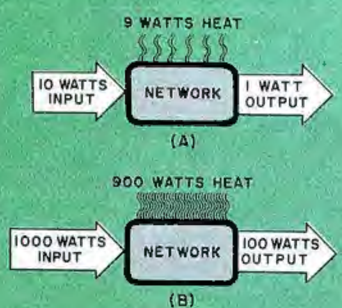

Fig. 1. In linear electronic networks, output power is proportional to input power.

Fig. 2. This drawing shows the cumulative effect of electronic attenuators placed in series.

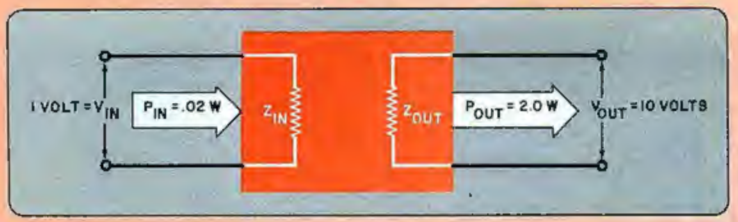

Fig. 3. Simple network illustrates voltage and power relationships.

use these relationships: $P_W = 10^{0.1\,P(dBw)}$; $P_{mW} = (10^3)\ 10^{0.1\,P(dBw)}$; $K = 10^{0.1G}$; where P(dBw) is the power in dBw, P_W is the power in watts, P_{mW} is the power in milliwatts, G the gain in decibels, and K the gain factor or power ratio. Similarly, $P_{mW} = 10^{0.1\,P(dBm)}$ and $P_W = (10^{-3})\ 10^{0.1\,P(dBm)}$ where P(dBm) is the power in dBm.

Moreover, it's also possible to use the tables in reverse. Multiplication of ratios can be accomplished by adding decibels. For example, 80 dB = 40 dB + 40 dB, so $K_{80\ dB} = (K_{40\ dB})\ (K_{40\ dB})$ or 1/100,000,000 = (1/10,000) times (1/10,000). Thus you can always break down a given number of decibels into several components that are listed in the tables. The same technique can be used for power levels in dBm: +80 dBm = +50 dB + 30 dB, and P_W = (100 watts) (1000) = 100,000 watts.

Decibels and Voltage Ratios. Expressing voltage ratios in decibels is also commonly done. The following relationship is used to compute the decibels of power gain of a voltage ratio—providing the network's input and output impedances are equal: $G(dB) = 20\ \log_{10}\ (V_O/V_I)$, where V_O and V_I are the rms output and input voltages, respectively. Keep in mind that the input and output impedances are assumed to be equal. This is often a valid assumption in r-f work because most circuit impedances are standardized at 50 ohms. It's not always true, however, and a disparity between the impedances can lead to incorrect values of decibel gain and confusion on the part of the person doing the calculations. Unless the impedances are known to be equal, it's probably better to stick to power ratios.

Here's a simple problem worked out both ways. What are the input and output power, gain factor, and decibel gain for the network shown in Fig. 3? First, we calculate signal power using the equation $P = E^2/Z$: $P_I = 1^2/50 = 1/50 = 0.02$ watt and $P_O = 10^2/50 = 100/50 = 2.0$ watts. Then $K = P_O/P_I = 2.0/0.02 = 100$ and $G(dB) = 10\ \log_{10}\ (K) = 10\ \log_{10}\ (100) = 10\ (2)$ or 20 dB. In the alternative solution, we use the voltage ratio expression: $G(dB) = 20\ \log_{10}\ (V_O/V_I) = 20\ \log_{10}\ (10/1) = 20\ \log_{10}\ (10) = (20)\ (1) = 20$ dB. Table III lists common voltage ratios and their resulting power gains in decibels—if input and output impedances are equal.

Applications in Communications. As mentioned earlier, an important property of decibels is their additive nature. The following real-life situation will illustrate how decibels simplify the solutions to fairly complex problems.

A radio amateur has a 2-meter trans-

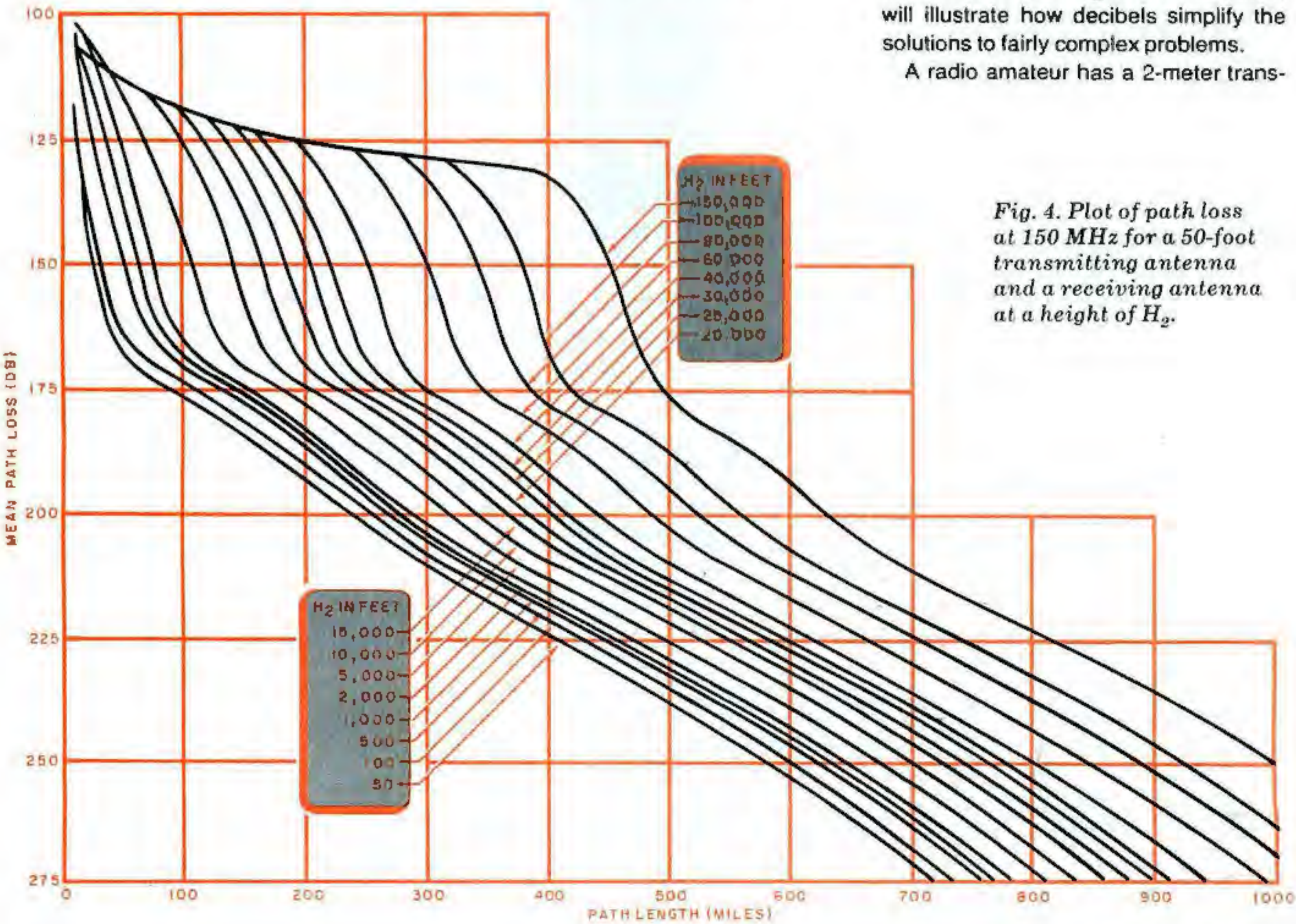

Fig. 4. Plot of path loss at 150 MHz for a 50-foot transmitting antenna and a receiving antenna at a height of H_2.

TABLE I—DECIBELS VS POWER RATIOS

Gain (dB)	Gain (power ratio)
−50	0.00001
−45	0.00003
−40	0.00010
−35	0.00032
−30	0.00100
−25	0.00316
−20	0.01000
−19	0.01259
−18	0.01585
−17	0.01995
−16	0.02512
−15	0.03162
−14	0.03981
−13	0.05012
−12	0.06310
−11	0.07943
−10	0.10000
−9	0.12589
−8	0.15849
−7	0.19953
−6	0.25119
−5	0.31623
−4	0.39811
−3	0.50119
−2	0.63096
−1	0.79433
0	1.00000
1	1.25893
2	1.58489
3	1.99526
4	2.51189
5	3.16228
6	3.98107
7	5.01187
8	6.30957
9	7.94328
10	10.00000
11	12.58925
12	15.84893
13	19.95262
14	25.11886
15	31.62278
16	39.81072
17	50.11872
18	63.09573
19	79.43282
20	100.00000
25	316.22775
30	1000.00000
35	3162.27744
40	10000.00000
45	31622.77222
50	100000.00000

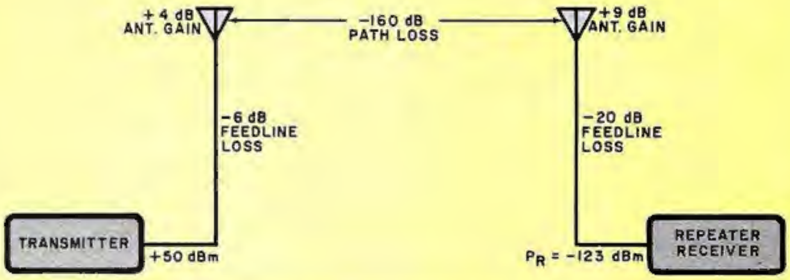

Fig. 5. Diagram showing the various gains and losses encountered in a communications system.

ceiver with 100 watts of r-f output. It is connected to a 5/8-wavelength antenna mounted on a 50-foot (15.2-m) tower via a 100-foot (30.5) length of RG-58A/U coaxial cable. He wants to work a repeater 90 air miles away whose 9-dB gain antenna is mounted on a 2000-foot (610-m) peak. The repeater requires −113 dBm of signal power at the input of its receiver for full quieting. The 2500-foot (762-m) length of low-loss coax interconnecting the repeater's antenna and receiver exhibits 20 dB of attenuation. Will his signal quiet the repeater? If not, what can be done about it?

Figure 4 is a plot of path loss at 150 MHz for a 50-foot (15.2-m) transmitting antenna and a receiving antenna at height H_2. It is taken from the "Transmission Loss Atlas for Select Aeronautical Service Bands from 0.125 to 15.5 GHz," by Gierhart and Johnson, available from the Superintendent of Documents, U.S. Government Printing Office, Washington, D.C. 20402, for $1.25. This graph tells us that for a 2000-foot high receiving antenna, a path length of 90 miles results in an antenna-to-antenna loss of 160 dB. Of course, this is only a nominal figure. The exact path loss will depend on terrain, ground conductivity, ground moisture, the weather, etc. Also, the quoted 160-dB path loss does not take antenna gain into account.

Referring to Table II, we see that 100 watts is +50 dBm. Also, we see from the ARRL Handbook that losses for RG-58A/U are approximately 6 dB per

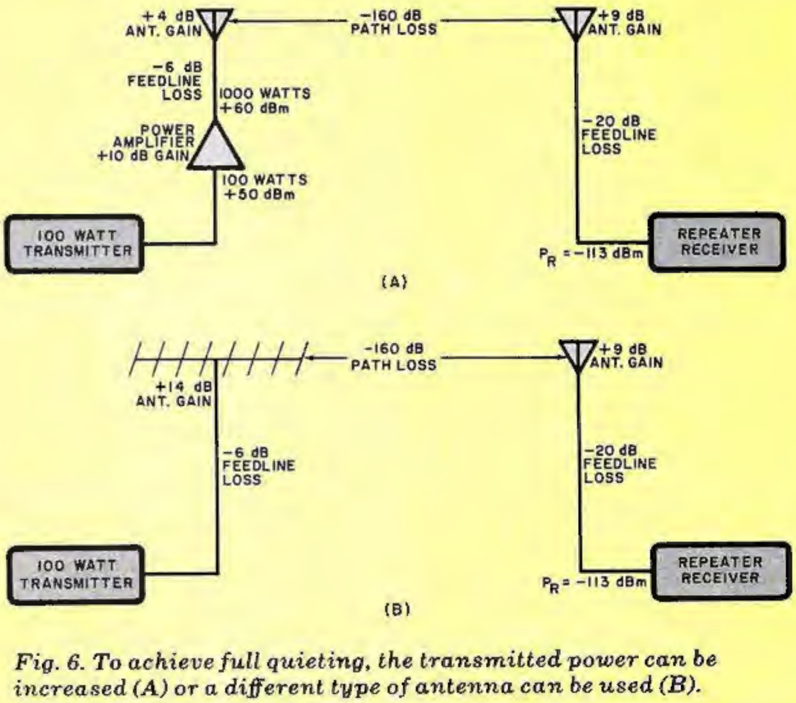

Fig. 6. To achieve full quieting, the transmitted power can be increased (A) or a different type of antenna can be used (B).

TABLE II—POWER IN dBm VS POWER IN WATTS AND MILLIWATTS

Power (dBm)	Power (milliwatts)	Power (watts)
−50	0.00001	0.00000001
−45	0.00003	0.00000003
−40	0.00010	0.00000010
−35	0.00032	0.00000032
−30	0.00100	0.00000100
−25	0.00316	0.00000316
−20	0.01000	0.00001000
−19	0.01259	0.00001259
−18	0.01585	0.00001585
−17	0.01995	0.00001995
−16	0.02512	0.00002512
−15	0.03162	0.00003162
−14	0.03981	0.00003981
−13	0.05012	0.00005012
−12	0.06310	0.00006310
−11	0.07943	0.00007943
−10	0.10000	0.00010000
−9	0.12589	0.00012589
−8	0.15849	0.00015849
−7	0.19953	0.00019953
−6	0.25119	0.00025119
−5	0.31623	0.00031623
−4	0.39811	0.00039811
−3	0.50119	0.00050119
−2	0.63096	0.00063096
−1	0.79433	0.00079433
0	1.00000	0.00100000
1	1.25893	0.00125893
2	1.58489	0.00158489
3	1.99526	0.00199526
4	2.51189	0.00251189
5	3.16228	0.00316228
6	3.98107	0.00398107
7	5.01187	0.00501187
8	6.30957	0.00630957
9	7.94328	0.00794328
10	10.00000	0.01000000
11	12.58925	0.01258925
12	15.84893	0.01584893
13	19.95262	0.01995262
14	25.11886	0.02511886
15	31.62278	0.03162278
16	39.81072	0.03981072
17	50.11872	0.05011872
18	63.09573	0.06309573
19	79.43282	0.07943282
20	100.00000	0.10000000
25	316.22775	0.31622775
30	999.99993	0.99999993
35	3162.27744	3.16227743
40	10000.00000	10.00000000
45	31622.77222	31.62277222
50	100000.00000	100.00000000

100 feet (30.5 m). Because signal losses and gains are additive (see Fig. 5), we can quickly compute P_R, the received signal power: P_R = +50 dBm − 6 dB + 4 dB − 160 dB + 9 dB −20 dB or −123 dBm. The repeater requires a signal strength of −113 dBm, so we see that we are 10 dB too low. Hence, the signal at the output of the repeater will be somewhat noisy.

Figure 6 illustrates two possible solutions to the problem. The first and most obvious is to increase the transmitted signal power. Because the signal power at the receiver is 10 dB too low, this means that the transmitter output must be increased from 100 to 1000 watts. Adding a kilowatt amplifier to the transmitting station, as shown in Fig. 6A, will raise the signal power at the repeater's receiver to −113 dBm.

The second solution to the problem involves replacing the 5/8-wavelength antenna with a directional yagi beam (Fig. 6B). A 12-element beam with a 3.5-wavelength boom will give about 14 dB of gain. The additional 10 dB over the 5/8-wavelength antenna will result in −113 dBm of signal power, and hence full quieting of the repeater's receiver.

Decibels in Audio. Anyone who wants to be conversant in the field of audio must be well versed in decibels. This is so because many of the key operating characteristics of the circuits and electromechanical transducers employed in high-fidelity applications are expressed in part or in whole using decibels. For example, the frequency response of a cartridge, speaker, or amplifier is specified as +*X*, −*Y* dB from (typically) 20 to 20,000 Hz. In the case of the cartridge, the reference employed is a certain output level in millivolts. For a speaker, the reference is the sound pressure level corresponding to the threshold of audibility (0 dB = 2×10^{-4} dynes/cm^2, 2×10^{-4} microbars, or 10^{-16} watts/cm^2.)

The power output of an amplifier is commonly specified in dBw, where 1 dBw equals 1 watt. For example, an amplifier which can provide 100 watts of continuous output power per channel can be rated as having an output of 20 dBw. Program source components such as turntables, tape decks and tuners have several decibel-related specifications. Of prime interest to any prospective purchaser is the signal-to-noise ratio (S/N) at the output of the program source. This is typically rated by driving the source to a reference output level, removing the input signal, and measuring the residual noise at the output. The decibel relationship between the reference output voltage and the residual noise is the component's S/N. For a program source to be considered one of high fidelity, it should have an S/N of 55 dB or more.

The relatively new IHF FM tuner standard specifies that signal strength in sensitivity ratings is to be expressed in dBf, where the reference is the *femtowatt* or 10^{-15} watt (0 dBf = 1 femtowatt). This was done to base sensitivity measurements on signal power, thus resolving the ambiguity caused by varying source impedances. For example, the same tuner could have an "old" IHF usable sensitivity of 2.0 μV into its 300-ohm antenna input *or* 1.0 μV into its 75-ohm input jack. Under the updated system, the tuner has a sensitivity of 11.2 dBf no matter which input and source impedance is used.

Decibels are so pervasive in the field of audio that a full appreciation of them is one mark of the true audio buff. Tape recordists especially must be comfortable with decibels. For example, when choosing a microphone, he must consider its sensitivity—its relative efficiency of converting acoustic energy into electrical energy. There are several methods of determining a microphone's sensitivity, which is usually expressed in dB below a specified reference level. The two types of ratings commonly used are the open-circuit voltage rating and the maximum power rating.

The open-circuit voltage technique measures the unloaded output of the microphone when driven by a reference SPL (for example, 1 microbar), compares it to a reference voltage (1 volt), and extracts the decibel relationship between the two. If an SPL of 1 microbar causes a microphone to develop 1 volt of output signal, its sensitivity is 0 dB. Practical microphones deliver much smaller output levels, with typical open-circuit voltage sensitivities varying from about −70 dB for dynamic moving-coil microphones to −37 dB for capacitive microphones with built-in preamplifiers.

The maximum power method involves connecting the microphone to a load equal to its internal (source) impedance, driving it with a reference SPL, and measuring the output power delivered to the load. The reference output power level is 1 milliwatt and the reference SPL is usually 10 microbars. Therefore, if a microphone driven by 10 microbars delivers 0.001 microwatt (10^{-6} milliwatt) into its optimum load, its sensitivity would be −60 dB referenced to 1 milliwatt or −60 dBm.

TABLE III DECIBELS VS VOLTAGE RATIOS

Gain (dB)	Voltage Ratio
−50	0.00316
−45	0.00562
−40	0.01000
−35	0.01778
−30	0.03162
−25	0.05623
−20	0.10000
−19	0.11220
−18	0.12589
−17	0.14125
−16	0.15849
−15	0.17783
−14	0.19953
−13	0.22387
−12	0.25119
−11	0.28184
−10	0.31623
−9	0.35481
−8	0.39811
−7	0.44668
−6	0.50119
−5	0.56234
−4	0.63096
−3	0.70795
−2	0.79433
−1	0.89125
0	1.00000
1	1.12202
2	1.25893
3	1.41254
4	1.58489
5	1.77828
6	1.99526
7	2.23872
8	2.51189
9	2.81838
10	3.16228
11	3.54813
12	3.98107
13	4.46684
14	5.01187
15	5.62341
16	6.30957
17	7.07946
18	7.94328
19	8.91251
20	10.00000
25	17.78279
30	31.62278
35	56.23413
40	100.00000
45	177.82793
50	316.22775

Volume Units. Many tape decks' record level meters are calibrated in "VU" as opposed to dB. Others have meters calibrated in dB. This may lead some to conclude that VUs are different from dBs. That, however, is untrue. Electrically speaking, a change in signal level of 1 VU is equivalent to a level change of 1 decibel.

A VU *meter*, however, has carefully controlled ballistic characteristics governing how the meter deflects upward from −20 to 0 VU and how much momentary overshoot will occur. It also has a specified input impedance (3900 ohms), is to be used with a 3600-ohm series resistor, has a defined scale (−20 to +3 VU), employs a particular type of rectifier, and is an average-responding meter. All of these characteristics have been chosen so that every true VU meter will respond to complex speech and musical waveforms in a consistent manner.

Few of the level meters found in consumer tape decks are true VU meters, even though most are average-responding level indicators and have a scale calibrated in "VU." As Julian Hirsch's Audio Reports usually indicate, these meters do not have the ballistic response of a true VU meter. Even so, they are useful level indicators.

A dB meter, on the other hand, need not have true VU dynamic characteristics. In fact, it is customary to mark the scale in decibels if the meter is a peak-responding indicator. The German standards organization, DIN, has established equally well-defined characteristics for peak-reading meters, but in consumer decks these, too, are often ignored by manufacturers.

Summary. Decibels are used to express power ratios. Voltage ratios can be related to power ratios if input and output impedances are known. Therefore, it's possible to express voltage ratios in decibels based on their equivalent voltage ratios. Power levels are commonly specified in decibels relative to a standard reference—usually one milliwatt, resulting in the unit dBm, or one watt, resulting in the unit dBw. Because decibels are the logarithms of ratios, they can be added to determine the cumulative effect of series connections of gain or loss blocks. In short, decibels are indispensible tools in electronic circuit and system analysis. ◇