

Countering the effects of vibration on board-and-chassis systems

A simple analysis tells the electronics designer whether the circuit boards in his system will need strengthening against vibration stress

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□ At some time in its life cycle, all electronic equipment encounters some form of vibration, at the very least when transported from manufacturer to customer. This vibration can cause fatigue failure in consumer systems, as well as in military or industrial systems, unless the electronics engineer has analyzed the design for its vulnerability to vibration stresses and built in an adequate safety margin against them.

The procedure is straightforward. It involves basic vibration theory, which is not normally part of the electronics curriculum. But "A guide to vibration analysis" on page 102 explains the concepts involved, and the rest of this article tells how to combine these concepts into simple design rules for application to a board-and-chassis package.

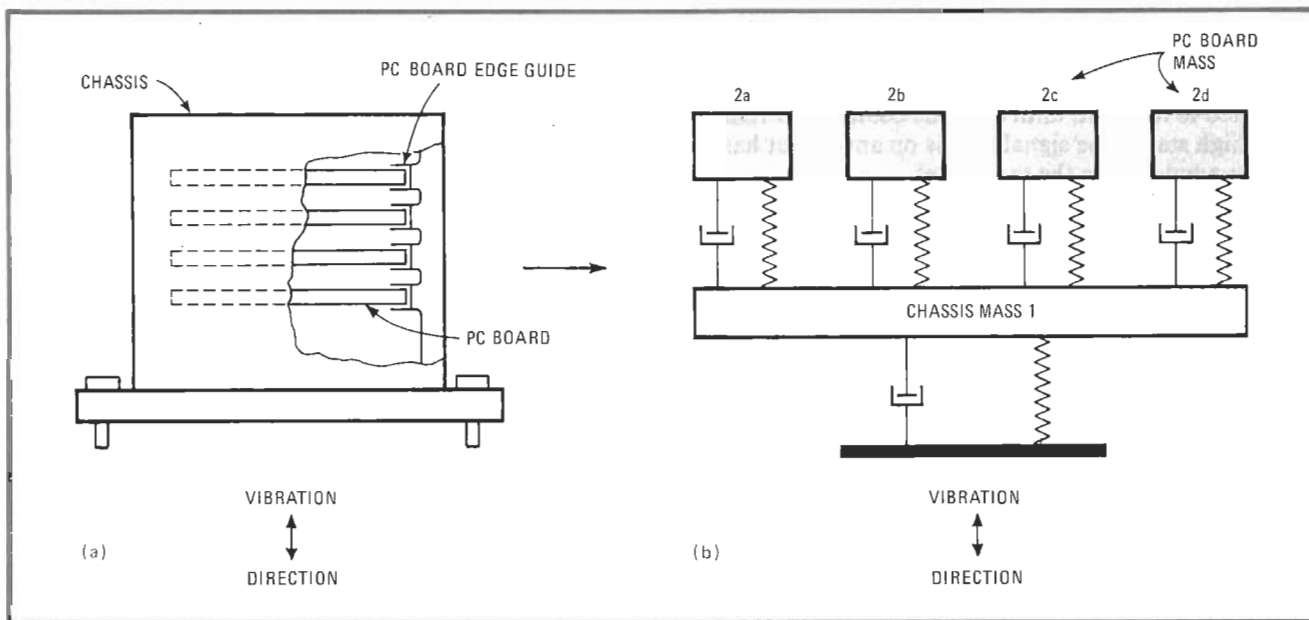
The basic building block of today's electronic system is the easily serviced, plug-in printed-circuit board. These boards are mainly rectangular, epoxy-glass units with plated copper wiring. In general, a pc board acts like a flat plate in a vibration environment and is subject to fatigue failures, particularly at board resonance. Vibration fatigue failures in these boards usually take

the form of broken leads, broken solder joints, and broken connectors. In extreme cases, parts have actually flown off a board under high vibration forces.

Applying the octave rule

Fatigue failures of these kinds can often be prevented by insuring that the boards and the chassis have different resonant frequencies. The goal is to avoid coincident resonances (see p. 102), which can amplify acceleration (g) forces in adjacent structural elements very rapidly. For example, if the resonant frequency of a chassis is too close to that of a circuit board within the chassis, high acceleration forces may develop in the board. High accelerations produce large deflections resulting in stresses that can culminate in rapid fatigue failures.

To illustrate, the chassis and the multiple plug-in boards of Fig. 1a will behave like a system capable of vibrating along more than one axis simultaneously—in other words, like a system with multiple degrees of freedom (see p. 102). The chassis usually represents the first-degree-of-freedom system for externally induced



1. Vibrating boards. A chassis with plug-in printed-circuit boards (a) has the equivalent mass spring analog of (b). The boards represent a second-degree-of-freedom system with respect to the chassis, since dynamic forces must be transmitted through the chassis to the boards.

vibrations. The boards represent the second degree of freedom with respect to the chassis since the dynamic forces must pass through the chassis before they reach the pc boards.

Coincident resonances can generally be avoided in multiple-degree-of-freedom systems by following the octave rule. This rule states that in a series spring mass system (see p. 102), the natural frequency of an element should be doubled for each additional degree of freedom in order to avoid severe resonant amplifications caused by a coincident resonance. Thus in Fig. 1b if the natural frequency of the chassis (mass 1) is 100 hertz, the natural frequency of each pc board (masses 2a, 2b, 2c, and 2d) should be at least twice that, 200 Hz or higher if possible. This separation of resonances prevents the chassis resonance from amplifying the pc board resonance, which in turn reduces the dynamic stresses and increases the fatigue life of the system.

Where to begin

What is the best starting point for a vibration analysis of the packaging configuration of Fig. 1? Should the natural frequency of the chassis or the pc board be determined first?

Experience, along with extensive analysis and testing, favors starting with the pc board. Tests have shown that it is possible to provide a long fatigue life (greater than 10 million cycles) for plug-in pc boards by basing the requirement for the maximum dynamic single-amplitude displacement (Y) of a rectangular board on the length of the shorter side (b) of that board. The displacement relation is shown by the equation:

$$Y_{\max} = 0.003 b \quad (1)$$

For example, the maximum dynamic single-amplitude displacement at the center of a rectangular pc board measuring 4 by 7 inches should be limited to a Y_{\max} of 0.003×4.0 in. or 0.012 in.

To provide a good fatigue life, Eq. 1 is based upon the dynamic stresses that are developed in the lead wires of the electronic components mounted on the board. As the board vibrates up and down during a resonant condition, it forces the electrical wires on the components to bend back and forth, as shown in Fig. 2.

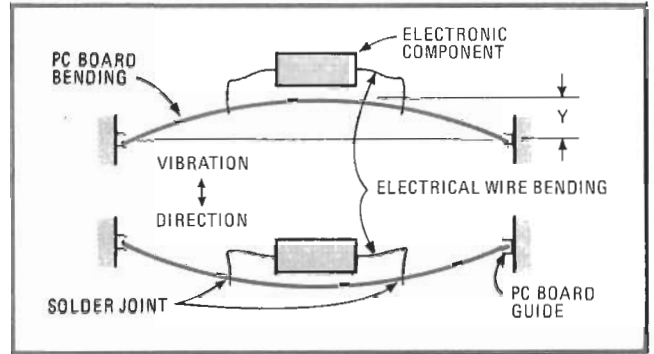
For a rectangular board, the most severe condition will occur when the body of the component is parallel to the shorter side of the pc board and at its center. The shorter side must have a more rapid change of curvature than the longer side because the displacement at the center is common to both.

The actual dynamic single-amplitude displacement of a single spring mass system can be determined from:

$$Y = 9.8 g Q / f^2 \quad (2)$$

where g is the acceleration force in gravity units, f is the sinusoidal vibration frequency, and Q is the transmissibility of the vibration from one element to the next (see p. 102).

Vibration test data on plug-in types of pc boards has shown that they act very much like a single-degree-of-freedom system when they are vibrating at their natural or fundamental resonant frequency. This test data shows



2. Over the waves. During a resonant vibration, a printed-circuit board moves up and down. These excursions can break leads and solder joints and even cause components to fly off.

that the approximate transmissibility for many different types of pc boards can be determined from this frequency, f_n :

$$Q = A (f_n)^{1/2} \quad (3)$$

where A is a dimensionless empirical constant dependent on the natural frequency, f_n , and acceleration inputs.

A large number of factors influence the transmissibility of a plug-in board. These factors will vary from one manufacturer to another because they each use different construction methods. Other variations include the component size, type of edge guides, type of electrical plug-in connector, conformal coating, and acceleration force.

Given acceleration inputs that vary from about 3 to 10 g, the value of A in Eq. 3 appears to be about 1.0 for pc boards with resonant frequencies between 100 and 400 Hz, drops to about 0.70 for boards with lower resonant frequencies between about 50 to 100 Hz, and rises to about 1.40 for those with resonant frequencies between about 400 to 700 Hz.

Equations 1, 2, and 3 can be combined to establish the minimum natural frequency required by a pc board for a long vibration fatigue life as:

$$f_n = (9.8 g A / 0.003 b)^{2/3} \quad (4)$$

As an example of how to use this equation, suppose a typical plug-in printed-circuit board is required to have a long fatigue life—10 million fatigue cycles at its resonant frequency. Suppose also that the board has 5-by-8-inch dimensions and must operate in an electronic system that will be subjected to a prolonged sinusoidal vibration environment of 6.0 g peak over a frequency range of 100 to 1,000 Hz. What needs to be determined is the minimum required pc board natural frequency plus the maximum allowable chassis natural frequency.

Start by assuming the value of A is 1.0, to see where the pc board resonant frequency will be. From the above data, $g = 6$ and $b = 5$. Substituting these values in Eq. 4, the minimum required pc board resonant frequency becomes:

$$f_n = \left[\frac{9.8 \times 6.0 \times 1.0}{0.003 \times 5.0} \right]^{2/3} = 248 \text{ Hz}$$

Since the board's resonant frequency is between 100 and

A guide to vibration analysis

Any discussion of vibration will involve references to degrees of freedom, transmissibility, coincident resonances, and series spring mass systems.

The **degrees of freedom** of a vibrating system describe the coordinates necessary to locate the position of the vibrating element at any time. For example, a single-degree-of-freedom system can move along only one axis, in both directions. A two-degree-of-freedom system will require two coordinates to describe the position of the elements. A multiple-degree-of-freedom system generally has many elements that can move along many axes.

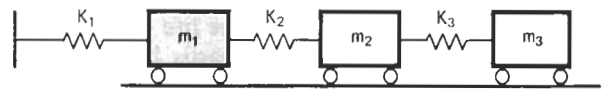
The **transmissibility** of a vibrating system is usually defined as the ratio of the maximum output force divided by the input force at a given frequency, or the maximum output displacement divided by the input displacement at a given frequency. In a lightly damped system, the maximum output force can easily be 100 times the input force. Such a system would have a transmissibility of 100. If a printed-circuit board has a maximum vibration input displacement of 0.001 inch at its edges at a frequency of 180 hertz, and the center of the board has a maximum vibration of displacement of 0.100 in. at its center at the same frequency, then the board has a transmissibility of 0.100/0.001 or 100.

A **coincident resonance** is a condition where two systems are joined together (or coupled) and both systems are vibrating near the resonant frequencies at the same time. It often happens that the amplified output from the first system turns out to be the input to the second

system, which amplifies that input a second time. If the transmissibility of the first system by itself is about 50, and if the transmissibility of the second system by itself is about 50, the joint (or coupled) transmissibility of the second system can approach about 50×50 or 2,500. If this happens in a real system, and it often does, the fatigue life is very short.

A **series spring mass system** consists of several springs and masses attached to one another, in a string-like manner. Any force or motion in the outermost member must pass through each adjacent member until it reaches the support. The figure shows a series system with three springs and three masses.

Of course, an electronic box does not really have springs and masses bouncing around during vibration—the springs and masses are used only as mathematical models, to simulate a system that will have vibration characteristics similar to the box. The mathematics associated with a spring and mass system is relatively simple compared to the mathematics required to analyze a complete box. The simplified mathematics permits a quick evaluation to be made of the structure supporting the electronics, to see how well it will hold up under the pounding it receives in a vibrating environment.



400 Hz, the assumed value of 1.0 for A is valid.

The natural frequency of the chassis that supports the pc board must also be established in order to prevent higher transmissibilities from developing in the board. Following the octave rule, the maximum natural frequency of the chassis must not exceed one half the natural frequency of the pc board, in this case, 124 Hz.

Finding that the pc board's minimum resonant frequency should be 248 Hz is only half the solution. It is still necessary to determine its actual natural frequency. Knowing the weight and thickness of the board of Fig. 3, it is possible to determine board natural frequency:

$$f_n = \frac{\pi}{2} \left[\frac{D}{\rho} \right]^{1/2} \left[\frac{1}{a^2} + \frac{1}{b^2} \right] \quad (5)$$

where D is the plate stiffness factor in pound-inches, ρ is the mass per unit area of pc board, a is board length, and b is board width.

The equation for plate stiffness factor is:

$$D = Eh^3/12(1-\mu^2) \quad (6)$$

where E is the epoxy Fiberglass modulus of elasticity, h is board thickness, and μ is Poisson's ratio for the board. In this example, $E = 2.0 \times 10^6$ lb/in.², $h = 0.090$ in., and $\mu = 0.12$, so that:

$$D = (2.0 \times 10^6 \times 0.090^3) / 12(1 - 0.12^2) \text{ lb in.} \\ = 123.3 \text{ lb in.}$$

The equation for mass, per unit board area is:

$$\rho = W/gab$$

so that for the 0.5-lb, 5-by-8-inch board, $W = 0.5$ lb, $a = 8$ in., and $b = 5$ in., and acceleration due to gravity, $g = 386$ in./s²:

$$\rho = (0.5) / (386 \times 8 \times 5) \text{ lb s}^2/\text{in.}^3 \\ = 3.24 \times 10^{-5} \text{ lb s}^2/\text{in.}^3$$

Substituting the values of D and ρ in Eq. 5:

$$f_n = \frac{\pi}{2} \left[\frac{123.3}{3.24 \times 10^{-5}} \right]^{1/2} \left[\frac{1}{8^2} + \frac{1}{5^2} \right] \\ = 170.4 \text{ Hz}$$

Use of Eq. 5 results in a board natural frequency of only 170.4 Hz, which is below the minimum required natural frequency of 248 Hz. This design is not satisfactory, and its natural frequency must be increased. If pc board thickness is increased to 0.125 in., the natural frequency will be increased to 287.7 Hz, which is satisfactory. If for some reason the pc board thickness cannot be increased, then ribs can be added to stiffen the board. A single vertical rib 0.090 in. thick and 0.250 in. high can be made of epoxy Fiberglass and cemented across the center of the board parallel to the 5.0-in. dimension. This would raise the natural frequency to 275 Hz, creating a satisfactory solution for the vibration environment.

A design of this type might take perhaps one man-day, but it could eliminate many hours of time in the field spent servicing vibration-caused failures. \square

Bibliography

David S. Steinberg, "Vibration Analysis for Electronics Equipment," John Wiley & Sons, New York, N.Y., 1973.