Karnaugh Map Display

A low-cost instrument which may be used to assist in teaching logic and Boolean algebra

*by Brian Crank**

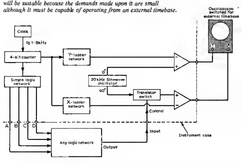

Fig. 1. A simplified block diagram of the instrument. Practically any oscilloscope will be suitable because the demands made upon it are small although it must be capable of operating from an external timebase.

Although this instrument employs the same basic principles as the earlier *Wireless World* Logic Display Aid (May to December 1969) any resemblance ends there. The present instrument is simpler and is very much cheaper. The circuit has been reduced to just four integrated circuits, three digital and one linear, and four transistors plus a few resistors, capacitors and diodes. The cost need not exceed a few pounds.

Mind you, the present instrument is not nearly so versatile as the earlier design although the display is more pleasing to the eye. The instrument will produce, on the screen of an oscilloscope, the Karnaugh map of any combinational logic circuit. If you are not completely familiar with Karnaugh maps a simple description will be found in the appendix to this article.

The reason for designing an instrument which will produce a Karnaugh map for any circuit is quite simple. The student is often taught Boolean algebra and logic through the use of the Karnaugh map. It completes the circle for the student to see a logic circuit producing the same map that was used to explain the operation of the circuit in the first place. In other words the theory and the practice can be brought closer together.

You may remember that in the earlier display the characters nought and one were displayed on the oscilloscope screen in the form of a pattern of dots. In the present design the characters are drawn as continuous lines exactly as you would draw them by hand. A typical display is shown in Fig. 11(a). Another advantage over the earlier design is that only two leads are needed between the instrument and the oscilloscope. These are the leads for X and Y deflection; an intensity modulation lead is not required.

As already mentioned the circuit is hybrid in that both linear and digital circuits are employed. Broadly the characters are positioned using a combination of both linear and digital techniques and the characters themselves are formed by linear circuits. The choice of whether to display a nought or a one at a particular position is taken by the logic circuit the Karnaugh map of which is to be displayed.

To use the instrument all one does is to connect it to the X and Y inputs of an oscilloscope, and connect any logic circuit to the instrument; the Karnaugh map for that circuit will then appear on the screen.

A block diagram of the instrument is given in Fig. 1. In brief, a clock pulse generator is used to drive a four-bit counter. The counter is split into two and each half drives a resistive ladder network. The ladder networks perform digital-to-analogue conversions and the resulting four-step staircase waveforms are fed to operational amplifiers which are used to drive the oscilloscope's X and Y deflection inputs. The oscilloscope is switched for external timebase operation. This produces sixteen dots on the screen arranged in a four by four matrix.

A sine wave oscillator, with a frequency much higher than the clock generator, produces two outputs which have a 90° phase difference. The sine wave corresponding to 0° is fed to the Y operational amplifier and the 90° waveform is fed to the X operational amplifier via an attenuator and a transistor switch. The result of the two sine waves on the screen of the oscilloscope is a vertical ellipse similar to the '0' printed here. The net result of both the sine and staircase waveforms is to display on the c.r.t. a four-by-four matrix of 0s. If the switch in the sine wave lead to the X operational amplifier is open there will be no horizontal sine wave component in the deflection waveform so on the screen will appear sixteen 1s. The 1 is formed by the sine wave input to the Y amplifier.

The counter that drives the ladder networks also drives a logic circuit which produces outputs that comply with the rules of a Karnaugh map. These outputs are used to drive the logic circuit you wish to display and the output of this logic circuit is used to control the 0/1 switch at the input to the X operational amplifier. Each section of the instrument will now be described in detail.

Sine wave oscillator

The sine wave oscillator is used to produce a Lissajous figure which represents 0 in the display and to do this, as we have already seen, it must produce outputs at 0° and 90°. An early version of the instrument used a sine wave RC oscillator followed by a 90° phase-shift network. Although this worked it was unsatisfactory because it was necessary to specify close tolerance components for the frequency

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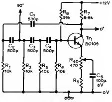


Fig. 2. The four-section phase-shift oscillator used to produce the characters which form the display. Operating frequency is about 22kHz.

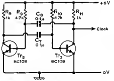


Fig. 3. Astable multivibrator clock generator which runs at about 1.4kHz.

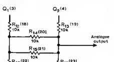


Fig. 4 The Y ladder network. Component reference numbers in brackets refer to the X ladder. The circuit converts the output of a counter into a staircase waveform by performing a digital-to-analogue conversion.

to be right for the phase shift required. An *LC* oscillator could have been used with the advantage that the frequency adjustment, to line the oscillator up with the phase-shift network, would have been no problem. However, coils, as well as being fairly bulky at the frequency we are interested in, are not the most popular items in constructional articles so it was decided to find a solution using *RC* circuitry.

The circuit employed is shown in Fig. 2. As can be seen it is a single transistor phase-shift oscillator. Normally a phase-shift oscillator employs three *RC* sections, each section phase shifting by 60°, to obtain the 180° phase shift necessary to obtain positive feedback and oscillation.

In the present design four *RC* sections are employed, each section shifting by 45° (4 × 45° = 180). It is now a simple matter to pick off the 90° signal after two 45° phase shifts at the output of the second *RC* section.

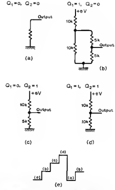


Fig. 5. The equivalent circuits of the ladder network for the four different conditions of the counter driving it.

The potentiometer R_{40}, the only adjustment in the whole instrument, is used to vary the a.c. gain of Tr_1 while maintaining d.c. conditions. The gain must just be enough to overcome the losses in the phase-shift network. If the gain is too low oscillation will not occur; if it is too high distortion will result. Potentiometer R_{40} is adjusted for a good sine wave output from Tr_1. The frequency of oscillation is about 22kHz but this is not at all critical.

Clock generator and counter

The clock generator is shown in Fig. 3. Little need be said about it as it is a conventional astable multivibrator which runs at about 1.4kHz.

The four-bit counter is formed by one t.t.l. (transistor-transistor logic) integrated circuit type SN7493N. This i.c. comes in the m.s.i. or medium scale integration class. It contains four *J-K* flip-flops and is connected as shown in the main circuit diagram (Fig. 10). The four flip-flops are cascaded to form a standard binary counter.

Looking at only the first two flip-flops, the outputs of which are called Q_1 and Q_2, the following outputs are produced:

Q_2	Q_1
0	0
0	1
1	0
1	1

The outputs of the second pair of flip-

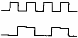


Fig. 6. Shows how the output of the t.t.l. binary counter is affected by the clock generator. The steps in the waveform are removed by a clamping circuit.

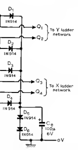


Fig. 7. The clamping circuit. The outputs to the ladder networks are the voltage drops across three forward-biased diodes in series.

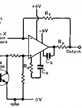


Fig. 8. The X deflection amplifier complete with the single-transistor 1/0 switch. The Y deflection amplifier circuit is the same but Tr_4 and its associated components are omitted.

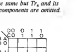


Fig. 9. Karnaugh map edge coding. A graticule, the same as this drawing, should be made so that the display on the c.r.t. can be viewed through it.

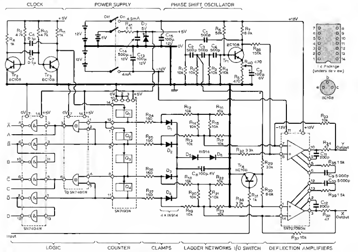


Fig. 10. The complete circuit of the Karnaugh map display instrument. The SN7486N actually contains four exclusive-OR gates, however, only two are used here.

flops, Q_3 and Q_4, produce exactly the same output but at one quarter of the frequency.

Ladder networks

The two ladder networks are connected to the binary counter. When the flip-flop Q_1 is at 0 the Q_1 output is connected, via a saturated transistor, to the 0V line. When the output Q_1 is at 1 it is connected via a saturated transistor to +6V.

The circuit of one ladder network is given in Fig. 4. The inputs Q_1 and Q_2 are switched according to the table given earlier. So Fig. 4 can be redrawn for each of the four states of the counter, so far as the output voltage is concerned (Fig. 5). If you would care to do the sums you will find that the output will rise from 0V in equal steps to produce a staircase.

Clamping network

Unfortunately the output of the flip-flops is not a good square wave. Although the rise and fall times are far more than adequate for the instrument the output of a particular flip-flop is affected by its input conditions. Fig. 6 illustrates this point.

The step in the waveform causes a corresponding step in the output of the ladder network which in turn causes certain characters on the display to appear double. The cure for this trouble is to add a clamping network which slices the top off the output from the flip-flops. This network is shown in Fig. 7.

The diodes $D_{1 \text{ to } 4}$ isolate the outputs of the flip-flops from each other and resistors $R_{24 \text{ to } 27}$ limit the current to a safe value. The output to the ladder network is now the voltage drop across three diodes in series.

Operational amplifiers and 1/0 switch

The well known operational amplifier type 709 is used in the instrument. The particular version employed (SN72709DN) is manufactured by Texas Instruments and includes two 709 amplifiers in a single dual-in-line package. The circuit of the X deflection amplifier is shown in Fig. 8. The Y deflection amplifier is identical except that the 1/0 switching transistor, Tr_4, and its associated components are omitted.

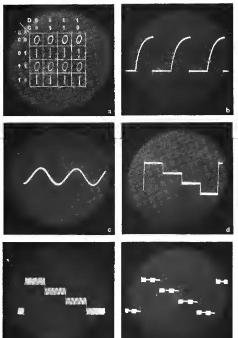
Resistors R_p and R_f combine to form the feedback resistor which sets the overall gain of the amplifier. Additionally R_p protects the amplifier from accidental short circuit of the output leads by limiting the output current. R_d, C_a and C_b are frequency compensation components which ensure stability.

The BC108 (Tr_4) is the switch which is controlled by the external logic circuit. It short-circuits the 90° output of the sine wave oscillator to ground when a 1 is required on the c.r.t. R_k is of a sufficiently large value to prevent the switch from significantly affecting the oscillator itself.

Logic circuit

Imagine that the Karnaugh map of Fig. 9 is superimposed on the c.r.t. face. Because of the action of the previously discussed circuitry the c.r.t. spot first rests in the top left-hand square, it then moves to the next square down, then to the square below that until it reaches the bottom of the column. The spot then flies back to the top but this time to the second column. The process continues until all 16 squares have been scanned. The spot then goes back to the first square again and the process is repeated, such is the effect of the two staircase waveforms. Each square on the map corresponds to a particular state of the counter. For instance, the top left-hand square is scanned when the counter outputs are all 0, that is at the top of both staircase waveforms (both the X and Y amplifiers invert).

We also know that each square on a Karnaugh map corresponds to a particular set of variables as defined by the coding at the edge of the map (see appendix if necessary). We must ensure that when the spot is in a particular square that the set of variables represented by that square are available at the output of the instrument for



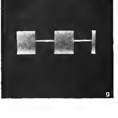


Fig. 11 (a). This is a photograph of the display which shows the map produced by an exclusive-OR gate connected to the A and B outputs ($\bar{A}$ B + A $\bar{B}$). The photograph has been 'doctored' in that the squares and edge coding have been drawn in. Normally this information would be contained on a graticule as shown in Fig. 9, however, this would be very difficult to photograph. The remainder of the photographs are waveforms within the unit. (b) 1.4kHz clock waveform, (c) sine wave oscillator (22kHz) taken at the emitter of Tr_1; (d) the staircase waveform at the output of the Y operational amplifier, for this photograph the sine wave oscillator was disabled, (e) Y deflection output when the display at (a) is being produced and (f) X deflection waveform under the same conditions; (g) waveform at the collector of Tr_4 when the display at (a) is being produced.

feeding to the external logic circuit. We must therefore compare the output of the counter with the Karnaugh map edge codings and rectify any differences that occur.

Karnaugh map edge coding		counter outputs	
B	A	Q_2	Q_1
0	0	0	0
0	1	0	1
1	1	1	0
1	0	1	1

The above table compares the output of the Y counter with the map's A B edge coding. The last two terms are different and therefore some logic is necessary to correct this.

Firstly on examination we can say that $Q_2 = B$ so a direct connection from the counter output Q_2 will form the output variable B.

Also, on examination, it can be seen that:

$$A = Q_1 . \bar{Q}_2 + \bar{Q}_1 . Q_2$$

which is our old friend the exclusive-OR function. We have already stated that the X counter outputs, Q_3 and Q_4, have the same outputs as Q_1 and Q_2 but at a slower rate and we can see that the Karnaugh map coding for C and D is the same as for A and B. We must therefore conclude that an identical logic function is required, namely

$$D = Q_4$$

and $C = Q_3 . \bar{Q}_4 + \bar{Q}_3 . Q_4$

The circuit of the logic section of the instrument can be seen on the lower left-hand side of the main circuit diagram, Fig. 10, and it can be seen that only two integrated circuits are required. The output variables, A, B etc., are buffered by simple inverters to prevent external connections from upsetting the operation of the counter. These inverters also provide the complement of the variables, $\bar{A}$, $\bar{B}$ etc.

Complete circuit

Fig. 10 combines all the circuits discussed so far and therefore little need be said about it. The various waveforms present for a particular display are shown in Fig. 11. Because the sine wave oscillator and the clock are not synchronous flyback between characters takes a different route every time and is not visible on the screen at normal brightness levels. Because of this blanking (a Z connection to the oscilloscope) is not required.

Construction

Making the unit is quite straightforward and no special precautions need be taken. A photograph of the layout employed in the prototype is given in Fig. 12; several components will not be found in this picture because they are mounted on the reverse side of the board.

It is important to connect pins two and three of the binary counter (SN7493N) to the 0V line. These pins are inputs to a gate which resets the counter. If this is not done the counter will be held at 0000 and the unit will not function. The only adjustment is R_{40} which must be set to give a nicely shaped 0. If you wish to adjust the size of the characters changing the value of R_{28}

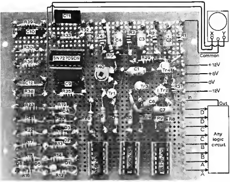


Fig. 12. A photograph of the prototype showing component positions. It should be noted that some parts have been mounted on the reverse side of the board and are therefore not marked. The integrated circuits are plugged into dual-in-line sockets which makes for easy removal.

will alter the height and R_{30} will alter the width.

Appendix

Karnaugh maps: The Karnaugh map is a means of pictorially showing all possible combinations of a number of two-state variables. Because of the way it is constructed it has other properties which make it possible to simplify Boolean expressions with the minimum of effort although it must be said that for more than four variables it is usually better to employ a more advanced method.

We will construct a Karnaugh map for four variables. The map will be the same as that displayed on a c.r.t. using the instrument described in the article. The basis of a Karnaugh map is a square. Each variable (usually labelled A, B, C and D for convenience) is allocated half the area of the square. To indicate the area occupied by a particular variable a simple edge coding system is employed. Fig. 13(a) shows the area occupied by the variable A and it is the area adjacent to the 1s under A in the edge coding. What is the area adjacent to the 0s under A in the edge coding? This is obviously the area representing $\bar{A}$. If the square of Fig. 13(a) is cut out and rolled into a cylinder the areas representing A and A become continuous—but more about that later. In Fig. 13(b) the areas representing B and $\bar{B}$ have been added. The square is now divided in four and each section represents one of the four possible combinations of A and B. From top to bottom, reading the edge coding, the sections are $\bar{A}\,\bar{B}$, $A\,\bar{B}$, $A\,B$, $\bar{A}\,B$.

You may have noticed that as you progress down the map, or up for that matter, only one of the variables alters at a time and this still applies if the map is rolled into a cylinder again because $A\,B$ becomes adjacent to $\bar{A}\,\bar{B}$.

In Figs. 13(c) and (d) the variables C and D have been added. If you consider only these two variables and roll the map into a cylinder the opposite way each section differs by only one variable. Reading round the tube so formed we get $C\,\bar{D}$, $C\,\bar{D}$, $C\,D$, $\bar{C}\,D$, $\bar{C}\,\bar{D}$ etc.

Looking at the map as a whole it is plain to see that each one of the sixteen squares we have formed represents one of the possible combinations of the four variables. For instance the top left-hand square, as can be seen by the edge coding, represents $A\,\bar{B}\,\bar{C}\,\bar{D}$ and the bottom right hand square represents $\bar{A}\,B\,\bar{C}\,D$.

But more important still is that adjacent squares, horizontally or vertically not diagonally, differ only in the negation of one of the variables. We have also proved, by rolling the map into a cylinder, that the top of the map is adjacent to the bottom and the left-hand edge is adjacent to the right-hand edge.

Two simple examples will show how these properties can be used to simplify Boolean expressions. Consider the expression $A\,\bar{B}\,C\,\bar{D} + A\,\bar{B}\,C\,D$. Draw a map as in Fig. 13(d) and put a 1 in the two squares representing the terms in the expression and an 0 in all the other squares. Because the 1s are adjacent to one another they are ringed as shown in Fig. 13(e). The simplified expression is derived by taking only variables which are common in adjacent terms. So $A\,\bar{B}\,C\,\bar{D} + A\,\bar{B}\,C\,D$ reduces to $A\,\bar{B}\,C$.

Fig. 13(f) shows the Karnaugh map for the expression $\bar{A}\,\bar{B}\,\bar{C}\,\bar{D} + \bar{A}\,\bar{B}\,\bar{C}\,D + A\,B\,\bar{C}\,\bar{D} + \bar{A}\,B\,C\,D$. All terms are adjacent and form a square of their own so only variables common to all four terms need be used. Therefore, from the map of Fig. 13(f):

$$A\,\bar{B}\,C\,\bar{D} + \bar{A}\,\bar{B}\,\bar{C}\,D + \bar{A}\,B\,C\,\bar{D} + \bar{A}\,B\,\bar{C}\,D = \bar{A}\,\bar{C}$$

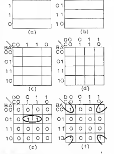


Fig. 13. The construction of a Karnaugh map and two examples. See text for full explanation

This brief explanation will serve to give the reader some idea of what a Karnaugh map is all about.

Next month a memory unit will be described which can be used with the Karnaugh map display unit, in place of the external logic circuit, to form an 'electronic blackboard'. Up to two Karnaugh maps can be stored, displayed or amended at will.

Shopping List

Resistors

All resistors, except the potentiometer, are 0.25W 5%.

10kΩ	× 18	150	× 4
470Ω	× 1	150k	× 1
56kΩ	× 1	33k	× 1
6.8kΩ	× 2	3.3k	× 2
1kΩ	× 3	47	× 2
4.7kΩ	× 2	1.5k	× 2
6.8Ω	× 1		

470Ω preset potentiometer.

Capacitors

500p	× 4	5,000p	× 2
100μ, 6V	× 2	200p	× 2
0.1μ	× 2	100μ, 12V	× 2
		500μ, 12V	× 1

Semiconductors

SN7493N, 4-bit binary counter,	× 1
SN7486N, quad exclusive-OR gate,	× ,,
SN7404N, hex inverter,	× ,,
SN72709DN, dual op-amp,	× ,,
BC108 transistors,	× 4
1N914 diodes,	× 6
5V, 400mW zener diode	× 1

Miscellaneous

dual-in-line sockets,	× 4
Lektrokit board type LK141,	× 1
Lektrokit pins,	× 100