GEAR DESIGN: Breaking the status quo

Traditional gear design limits the performance of mechanical drives.

Alex Kapelevich
President
AKGears
Shoreview, Minn.

Direct gear design optimizes tooth profiles and significantly reduces contact and bending stresses. This improves load capacity, reduces size and weight, improves efficiency, and extends life. The method works for gears with symmetric and asymmetric teeth.

Invention of the gear-hobbing process about 150 years ago revolutionized not only gear fabrication, but also gear design. Basically, a hob or shaper cutter traverses the gear blank to generate teeth. The advantage is that one gear-generating tool can machine gears with different numbers of teeth. And gears made by the same tool work together, which provides interchangeability and low tool inventory.

Hobbing also simplifies gear design. Parameters such as diametral pitch or module, profile angle, addendum, whole depth, and fillet-radius proportions define the cutting edge of the hob, called the generating gear rack. Over the years, these dimensions were standardized and became design parameters. Thus, 14.5°, 20°, and other basic gear racks serve as a starting point and foundation for gear design. Engineers need only calculate the diametral pitch or module for an operating load and then let the standard basic rack define all gear features. For better performance (higher efficiency, load capacity, and so on), standards recommend addendum modifications (X-shifts). It sounds like a near-perfect system.

However, this makes gear design indirect, as gear geometry depends on preselected, typically standard, basic gear racks. Times have changed. There is a clear trend toward custom gears that outperform standard versions, and interchangeability is often not critical.

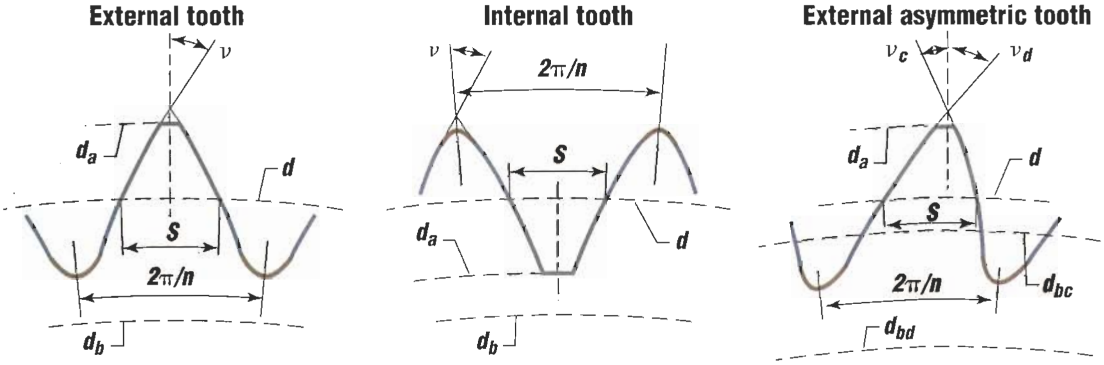
Low tooling inventory is also not that important. Regardless of the manufacturing method, modern gears require many dedicated cutting and clamping tools. High-performance gears — aerospace and automotive, for example — do not use standard basic racks for design. Instead, they rely on custom racks which provide, for instance, higher pressure angles or contact ratios.

And custom gears are more economical than ever. Manufacturing advances mean gear machining is no longer limited to hobbing or shaper cutting. Profile cutting, grinding, and broaching are also widely used. And newer gear-forming technologies are gaining ground, such as precision forging, casting, extrusion, powder-metal processing, and plastic and metal-injection molding. None of these processes uses traditional rack-generating methods. However, engineers still generally design gears based on preselected racks.

An alternative method, called Direct Gear Design, does not use tooling-rack parameters. Instead, it defines gear shape based on required performance and operating conditions — as is the case with most other mechanical components.

Edited by Kenneth Korane

Gear-tooth profiles



Basic tooth profiles are shown with the fillet portion in red. Subscripts d and c are for the drive and coast flanks of the asymmetric tooth.

DIRECT GEAR DESIGN

The idea is not new. Centuries ago, engineers successfully defined gear geometry based on required performance and operating conditions. Then they made gear drives that matched this geometry. It is important to note that gear geometry was defined first and manufacturing processes and tool parameters were secondary. This is the essence of direct gear design and just the opposite of today's standard practice.

Direct Gear Design has been developed for involute gears and is based on the Theory of Generalized Parameters, (*Gears with improved characteristics,* E.B. Vulgakov, Mashinostroenie, Moscow, 1974.) It involves an application-driven gear-development process with emphasis on maximizing performance and cost efficiency without concern for predefined tooling parameters.

There is no need for a gear rack to describe the gear-tooth profile. Two involutes of the base circle, the arc distance between them, and tooth-tip circle describe the gear tooth. If two involutes belong to two different base circles, they form asymmetric teeth. Equally spaced teeth form the gear. The fillet between teeth does not contact the mating gear. However, this portion of the tooth profile is critical because it has the maximum bending stress concentration.

Here is the mathematical basis for direct design. Two or more gears with equal base-circle pitch can mesh. Calculate the operating pressure angle α_w and the contact ratio ε_α for external gearing from:

$$\alpha_w = \operatorname{arcinv}\left[\left(\frac{\operatorname{inv}\nu_1 + u\operatorname{inv}\nu_2 - \dfrac{\pi}{n_1}}{1+u}\right)\right]$$

$$\varepsilon_\alpha = n_1\left[\frac{\tan\alpha_{al} + u\tan\alpha_{a2} - (1+u)\tan\alpha_w}{(2\pi)}\right].$$

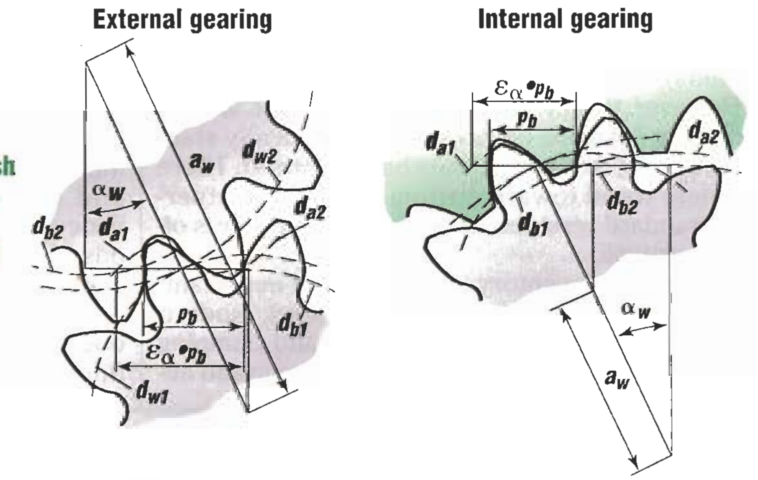
For internal gearing,

$$\alpha_w = \operatorname{arcinv}\left[\frac{(u\operatorname{inv}\nu_2 - \operatorname{inv}\nu_1)}{(u-1)}\right]$$

$$\varepsilon_\alpha = n_1\frac{[\tan\alpha_{al} - u\tan\alpha_{a2} + (u-1)\tan\alpha_w]}{2\pi}.$$

The gear ratio $u = n_1/n_2$; and the involute profile

Gear mesh



Significant parameters are shown here for two gears in mesh with equal base-circle pitch. Subscripts 1 and 2 are for the mating pinion and gear, respectively.

STRESS REDUCTION

Number of teeth		15	20	30	50	80	120
Bending stress reduction, %	Full radius 20° rack	6	10	10	10	10	10
	Optimized fillet	25	23	21	21	21	21

The table illustrates typical bending-stress reductions using a full-radius rack, and with a fillet profile optimized by direct design. Bending stresses are compared to those in a standard 20° rack for gears with different numbers of teeth. The involute portion of the tooth profile is the same. The full-radius rack reduces bending stress by about 10%. Fillet optimization reduces stress more than 20%. Complete tooth-profile optimization (involute and fillet) reduces bending stresses even more.

angle at the tooth tip diameter $\alpha_a = \arccos(d_b/d_a)$. The "inv" term refers to the involute function of an angle x, where $\mathrm{inv}x = \tan x - x$ and the angle is in radians.

For metric gears, the operating module $m_w = 2a_w/(n_2 \pm n_1)$. For English units, the operating diametral pitch $p_w = (n_2 \pm n_1)/2a_w$. The "+" symbol indicates external gearing; "–" is for internal gearing.

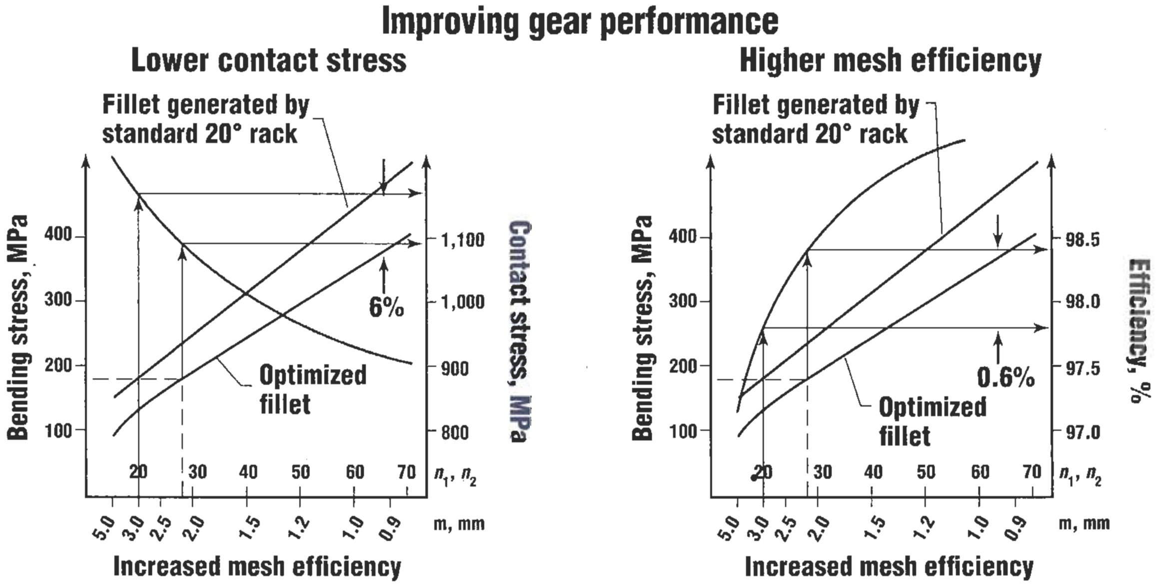
Engineers must define the tooth-fillet profile to completely describe nominal gear geometry. In traditional gear design, cutting-tool trajectory in generating motion defines the fillet profile. The most common way to reduce bending-stress concentration is by using a full-radius generating rack. In some cases, a parabola, ellipsis, or other mathematical curve forms the generating rack tip. These approaches have limited success in reducing bending stress, which depends on the generating rack's profile angle and number of gear teeth.

Direct design optimizes the fillet profile to minimize bending-stress concentration. The initial profile is a trajectory of the mating gear-tooth tip in the tight (zero-backlash) mesh. Then FEA and random-search methods refine the fillet to minimize bending stress. They also minimize radial clearance with the mating gear tooth while preventing interference for worst-case tolerances and operating conditions.

Direct design also maximizes curvature radius, evenly distributing bending stress along a large portion of the fillet and reducing stress concentration. The resulting profile depends on the mating gear geometry and differs for external and internal gears and gear racks. However, the profile does not significantly depend on load levels and application points. If the gear meshes with several different gears, such as the planet gear in a planetary stage, its optimized fillet profile prevents interference with any gear.

CUSTOM GEARS

Direct gear design is suited for all kind of involute gears, including spur, helical, bevel, worm, and face gears, as well as rack and pinions. Helical, bevel, and worm-gear tooth profiles are typically optimized at a section normal to the tooth line at the pitch circle. Face gear fillets vary in every section



The graphs present stresses and efficiencies of gears with constant center distance a_w = 60 mm; face width b = 10 mm; and driving torque T = 50 Nm — but with different fillets. Data show that for gears with the same bending stress level, those with an optimized fillet have finer pitch (smaller module) and more teeth. This reduces contact stresses and increases mesh efficiency. Gears with optimized fillets also have lower contact temperature, thicker hydrodynamic oil film between teeth, and higher wear resistance.

along the tooth line; therefore direct design optimizes the profile in several sections and then blends it into the fillet surface.

Gears with asymmetric teeth are naturally suitable for direct gear design because there are no standards for asymmetric generating racks. Asymmetric gear teeth improve performance in the primary drive direction by degrading performance in the opposite, coast direction. The coast tooth profiles are unloaded or lightly loaded during relatively short work periods.

The main advantage of asymmetric gears is that they reduce contact stress, resulting in higher torque density (load capacity per gear size). Also, direct design determines coast flanks of the teeth independent from the drive flanks, managing tooth stiffness while keeping a desirable pressure angle and contact ratio on the drive profiles. This reduces gear noise and vibration.

Direct-designed gears require custom tooling. For molding, forging, casting, and extrusion, tool-cavity profiles match the whole-gear profiles, adjusted for warpage and shrinkage. For profile cutting and grinding, tool profile is the same as a space profile between neighboring teeth.

NOMENCLATURE

a_w = Center distance
d = Reference circle diameter
d_a = Tooth tip circle diameter
d_b = Base-circle diameter
d_w = Operating pitch-circle diameter
n = Number of teeth
p_b = Base-circle pitch
S = Circular tooth thickness at reference diameter
α_w = Operating pressure angle
ε_α = Contact ratio
ν = Involute intersection profile angle

For gear-generating processing, the tooling-rack profile is based on gear geometry. The pressure angle, in this case, is selected to improve machining conditions. Generating and profile machining cannot form some fillet profiles. Direct design can adjust fillet profiles, trading stress reduction for better manufacturability.

Now designers can use traditional or direct gear design. Manufacturing concerns drive traditional gear design, where interchangeability, low tool inventory, and design simplicity are important. It is generally for lower-performance drives, such as off-the-shelf gears, low-volume machined gears, and drives with interchangeable gear sets.

COMPARING TRADITIONAL AND DIRECT DESIGN

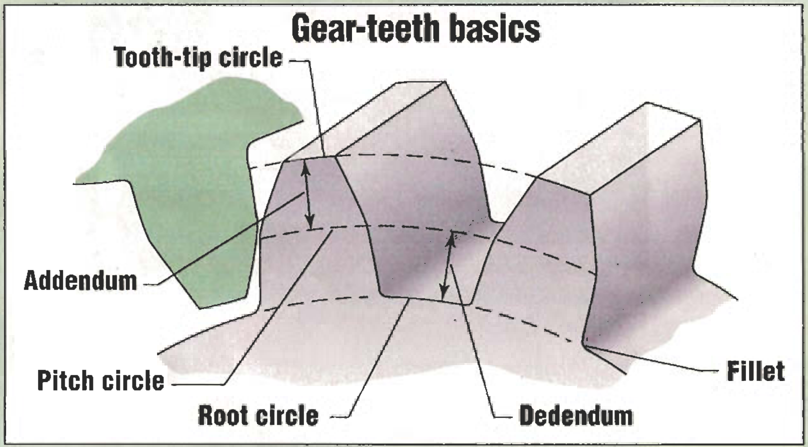
Design method	Traditional design with full-radius 25° rack		Direct gear design with complete tooth-profile optimization							
			Symmetric		Asymmetric		Symmetric, high-contact ratio		Asymmetric, high-contact ratio	
Gear	Pinion	Gear	Pinion	Gear	Pinion	Gear	Pinion	Gear	Pinion	Gear
Number of teeth	27	49	27	49	27	49	27	49	27	49
Module, mm	3.0		3.0		3.0		3.0		3.0	
Drive pressure angle, degrees	25		25		32		20		24	
Coast pressure angle, degrees	25		25		18		20		16	
Drive contact ratio	1.5		1.5		1.5		2.06		2.06	
Pitch diameter, mm	81	147	81	147	81	147	81	147	81	147
Tooth thickness, mm	4.98	4.34	4.98	4.34	4.98	4.34	4.98	4.34	4.98	4.34
Face width, mm	32	30	32	30	32	30	32	30	32	30
Driving torque, Nm	300	—	300	—	300	—	300	—	300	—
Contact stress, MPa	976		976		887		822		777	
Contact-stress difference, %	—		—		−9		−16		−21	
Bending stress, MPa	196	198	167	167	171	171	128	125	130	128
Bending-stress difference, %	—	—	−15	−15	−13	−13	−37	−37	−34	−35

This table presents data for a traditionally designed gear pair with bending stresses balanced by addendum modification, and generated by a full-radius 25° rack. Results are compared to similar gear pairs created by direct design for various tooth profiles: symmetric, asymmetric, and with conventional and high-contact ratios. This illustrates the potential benefits of complete tooth-profile optimization in terms of contact and bending stress reduction. AKGears has successfully used Direct Gear Design in more than 100 mechanical transmissions for aerospace, automotive, agricultural, medical, and other applications.

Direct gear design is application driven, when performance is critical. It is for custom gear drives and beneficial for mass-production, molded, forged, cast, and powder-metal gears. It maximizes gearbox performance for critical and extreme applications, such as in aerospace, automotive, and racing-gear transmissions.

MAKE CONTACT

AKGears LLC, *akgears.com*, (651) 308-8899.



A glossary of gear terminology

Gearing has a language all its own. Here's a quick review of some of the basic terms.

Addendum. The height of the tooth above the pitch circle.

Base circle. The imaginary circle used to generate the involute.

Center distance. The distance between axes of mating gears.

Dedendum. Radial distance between pitch circle and root circle.

Diametral pitch. The ratio of the number of teeth to the pitch diameter in 1/in.

Fillet profile. The curve that connects the involute tooth profile at the root area.

Full radius rack. A generating rack with full-radius addendum (the top land is zero).

Involute curve. A curve described by the end of a line unwound from the circumference of a circle.

Module. Pitch diameter in millimeters divided by the number of teeth.

Pitch circle. The circle which intersects the involute at the point where the pressure angle equals the profile angle.

Pressure angle. The angle between the line of action and a line tangent to the pitch circle.

Profile angle. Angle between a tangent to the involute tooth profile and the radius from the gear axis to the tangent point.

Reference circle. Can be any circle that intersects the involute tooth profile.

Tooth-tip circle. Circle that describes the tooth-tip lands.

Whole depth. The total depth of the tooth, the addendum plus the dedendum.