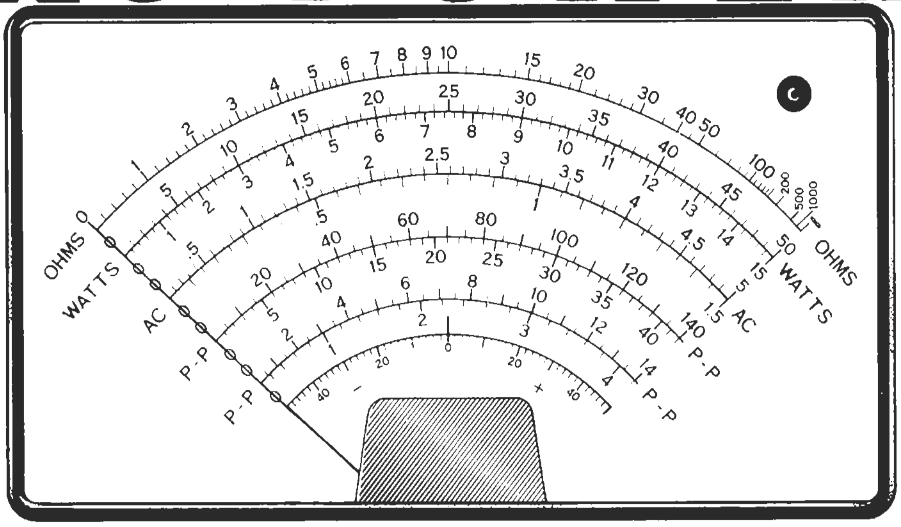
AC POWER MEASUREMENT



Learn the principles of measuring power.

HARRY L. TRIETLEY

AC POWER MEASUREMENT: YOU PROBABLY don't think about it very much. Most engineers and technicians normally don't even give it a second thought. Nonetheless, it's an extremely important and pervasive area of electronics, used in a wide variety of fields, include determining RF transmitter power and field strength, radar, motor and generator testing, sonar, stereo, and acoustics, and one area that affects all our daily. lives, commercial AC power distribution.

We're all billed using a kilowatt-hour (kWh) meter, and obviously want it to be accurate. Utilities need to measure true usage, because it affects generator loading and fuel consumption. Consumers likewise need to control operating efficiency, peak load usage, and billing. With rising fuel costs and the need for conservation, the subject has become increasingly relevant. AC power isn't always easy to measure, because it's affected by such factors as phase shift and waveform distortion. This article discusses techniques of measuring AC power, using some special IC's designed for this purpose.

True power and RMS voltage

Let's review DC power; the power law, $P = E \times I$, and Ohm's law, $E = I \times R$, produce the 12 equations in Fig. 1. They're for DC only, and omit capacitive and inductive reactances. They're valid for AC *if* the load is purely resistive (causing no phase shift), and *if* E and I are true *R*oot *M*ean *S*quare (RMS) values. RMS means *the square root of the average of the squares* of a series of voltages or currents. The RMS concept is necessary, because the time-average value of a sinusoidal voltage or current with no DC component is zero. Let's see why, and how this relates to power.

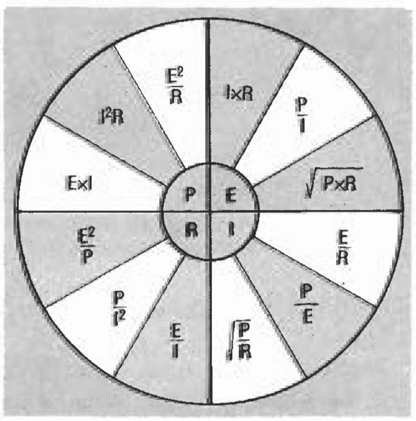


FIG. 1—OHM'S LAW AND THE POWER EQUATION combine to form 12 basic equations for resistive circuits.

Figure 2-*a* shows a sinusoidal voltage sampled 16 times, applied across R1 in Fig. 2-*b*. As each sample varies, the dissipated power is $P(t) = e^2(t)/R1$. To find the approximate RMS coefficient for time-average power, the power is found for each sample, and the average of the values is determined. Increasing the number of intervals increases resolution, and improves accuracy.

Table 1 shows the calculations; the average sampled instantaneous power is $P = 0.503 \times V^2_{PK}/R1 = (0.709 \times V_{PK})^2/R1$. The letters A–P in Fig. 2 correspond to the same letters in parentheses in the left-hand column of Table 1. The RMS value for

Table 1—DETERMINING THE AVERAGE AND RMS CONVERSION FACTORS

Sample Angle, deg	Sample Value	Sample Absolute Value	Square of Sample Value
11.25 (A)	0.195	0.195	0.038
33.75 (B)	0.566	0.566	0.320
56.25 (C)	0.831	0.831	0.691
78.75 (D)	0.981	0.981	0.962
112.25 (E)	0.981	0.981	0.962
134.75 (F)	0.831	0.831	0.691
146.25 (G)	0.566	0.566	0.320
168.75 (H)	0.195	0.195	0.038
191.25 (I)	−0.195	0.195	0.038
213.75 (J)	−0.566	0.566	0.320
225.25 (K)	−0.831	0.831	0.691
247.75 (L)	−0.981	0.981	0.962
281.25 (M)	−0.981	0.981	0.962
303.75 (N)	−0.831	0.831	0.691
326.25 (O)	−0.566	0.566	0.320
348.75 (P)	−0.195	0.195	0.038
Sums:	0.000	10.292	8.044

Average conversion factor: 10.292/16 = 0.643.
RMS conversion factor: 8.044/16 = 0.503, $\sqrt{0.503}$ = 0.709.

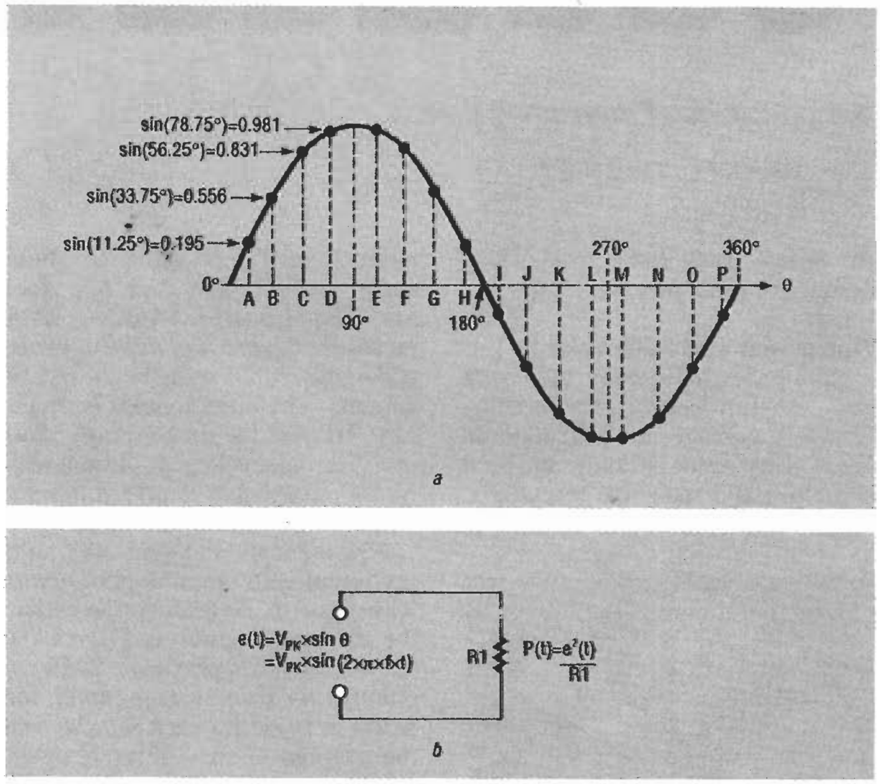


FIG. 2—A SINUSOID SAMPLED IN 16 EQUISPACED locations in (*a*) is applied across R1 in (*b*). This is the first step in the approximate calculation of the RMS conversion factor.

sinusoidal current is found similarly. If the RMS value for a different waveshape were desired, the process would be repeated using new samples. This crude approximation gives an RMS value of $0.709 \times V_{PK}$, versus a correct value of $V_{PK}/\sqrt{2} = 0.7071 \times V_{PK}$, an error of only 0.27461%.

The average or mean value *isn't* just the average of the actual sinusoidal samples, since this is 0 volts, as mentioned earlier. To find the average value, the *average of the absolute value* of the samples in Table 1 (the full-wave rectified case) is $0.643 \times V_{PK}$. The correct value is $0.6366 \times V_{PK}$, an error of only 1.01%. Fig. 3 shows the peak, RMS, and average values for several waveforms, with sketches of each.

One of the more difficult concepts to understand about power is that there are really two separate types of power, instantaneous and time-average. Since DC values are time-invariant, instantaneous and time-average DC powers are always equal. The term used when referring to an RMS value is that of "equivalency." RMS power, in an AC context, is considered "equivalent" to DC power. The major factor for judging this equivalency is what is referred to as the "I^2R" effect, from the power formula.

RMS power yields the same *heat dissipation* in a resistive load as an equal value of DC power. Reactive loads (those with inductance or capacitance) cause a phase angle difference between voltage and current equal to the phase angle of the load impedance. Such a load may have both resistive (real) and reactive (imaginary) components; real "I^2R" power (heat) dissipation occurs *only* in the *real* part, *never* the imaginary part. The imaginary part refers to the energy stored in capacitive electric fields, and inductive magnetic fields.

You should know the meaning of the word "imaginary" in this context; it refers only to the phase difference between voltage and current, *never* that these quantities aren't physically present. As you've probably heard before, "imaginary" voltages and currents are quite capable of causing real shocks.

Measuring true RMS voltage

You may have heard of "average-responding" and "true RMS" AC meters. Average-responding meters, shown in Fig. 4, are simpler; they rectify and filter AC to give the average value as a DC voltage, that's then scaled to display RMS. Thus, a sine wave with $V_{PK} = 1$ volt produces 0.6366 volt going into the display, but the display is calibrated to read 0.7071 volt. Because the relationship between RMS and average depends on the waveform, the reading is accurate *only* for sinusoids.

For example, average-responding meters aren't useful for measuring

RMS power in most modern circuits, since nonsinusoidal waveforms are very often used in SCR controllers and variable-speed motor drives, necessitating true RMS measurements. The most common methods are thermal, direct (explicit) computation, and implicit computation. In implementing these approaches, we'll also examine two RMS-to-DC converter IC's that take the work out of true RMS measurement.

Thermal RMS measurement

Thermal techniques find RMS voltage by exploiting the fact that the real component of power is dissipated as heat in a resistor, as mentioned earlier. In Fig. 5, R1 = R2; the measured voltage is amplified, buffered, and/or scaled by A1, and passed to R1. Similarly, R2 is connected to A2, a high-gain differential amplifier producing DC. Matched series-opposing temperature sensors, usually thermocouples, are in physical contact with R1 and R2, and A2 responds to their temperature difference. If R1 is hotter than R2, the output of A2 increases until the temperatures of R1 and R2 are equal. The two temperatures will be equal when the DC output of A2 equals the RMS output of A1.

Although this is a simple approach, accurate measurements using this method are difficult. The main problem is the need for highly accurate thermal matching. Any difference in size, shape, contact, or insulation causes errors. Also, thermal inertia has to be optimized. It has to be slow, to average the AC waveform over its whole cycle. However, excessive inertia causes sluggish measurements.

The Linear Technology LT1088 RMS-to-DC converter has matched pairs of 50- and 250-ohm resistors and diode sensors in a thermal package. The diode has a forward-bias voltage drop temperature coefficient of −1.75 mV per °C, and the two sets of internal resistors provide flexibility.

Figure 6 shows the LT1088 in a practical circuit. V_{IN} comes from an amplifier stage, heating the 50-ohm AC resistor while A1 heats the 50-ohm DC resistor via emitter-follower Q1. The level at pin 9 is 1 volt DC per volt of RMS input, while A2 provides gain adjustment. The maximum input is 4.25 volts for 50 ohms, and 9.5 volts for 250 ohms. Accuracy is 1% up to 50 MHz, with a 3-dB bandwidth for A1 of 300 MHz, and input crest factors (peak-to-RMS ratio) up to 50:1, making this method highly suitable for RF RMS measurement.

WAVEFORM	PEAK VOLTAGE	RMS VOLTAGE	AVERAGE VALUE
SQUARE WAVE	V_{PK}	V_{PK}	V_{PK}
25% DUTY CYCLE PULSE	V_{PK}	$0.5 \times V_{PK}$	$0.25 \times V_{PK}$
TRIANGLE WAVE	V_{PK}	$0.5774 \times V_{PK}$	$0.5 \times V_{PK}$
SAWTOOTH WAVE	V_{PK}	$0.5774 \times V_{PK}$	$0.5 \times V_{PK}$
SINE WAVE	V_{PK}	$0.7071 \times V_{PK}$	$0.6366 \times V_{PK}$
50% SINE (LIGHT DIMMER)	V_{PK}	$0.5 \times V_{PK}$	$0.3138 \times V_{PK}$

FIG. 3—PEAK, RMS, AND AVERAGE VALUES OF typical waveforms. Listed are a simple square wave (50% duty cycle, 0 volts DC), a periodic pulse train (25% duty cycle, positive-only), a triangle wave (0 volts DC), a sawtooth wave (positive-only), a sinusoid (0 volts DC), and a 50%-sine wave used in light dimmers.

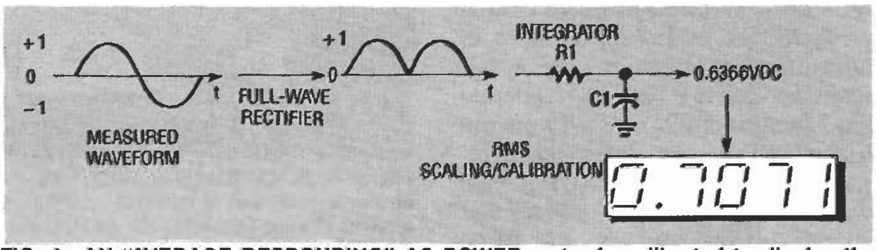


FIG. 4—AN "AVERAGE RESPONDING" AC POWER meter is calibrated to display the equivalent RMS voltage of a sinusoid only.

Direct RMS computation

Direct or explicit RMS computation solves the problem of RMS power measurement electronically. As shown in Fig. 7, the amplified or buffered AC is rectified, squared, averaged, and presented to a circuit that determines the square root of the average, giving the RMS level. The problem here is the dynamic range of the squared signal.

The squaring multiplier produces full-scale output with both X and Y at full scale. Thus, 100% inputs produce 100% output, 10% inputs produce 1%

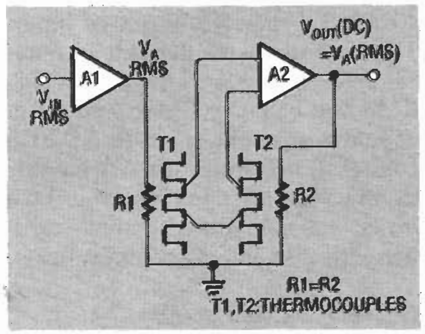


FIG. 5—THERMAL RMS VOLTAGE MEASUREMENT by the power dissipated by R1 and R2. Here, R1 = R2; matched temperature sensors contact R1 and R2, and A2 responds to the temperature difference. If R1 is hotter than R2, the A2 output increases until the resistor temperatures are equal, and the DC voltage across R1 equals the RMS voltage across R2.

output, and 5% inputs produce 0.25% output, and so on. Thus, this method isn't suited to voltages varying over a wide range, or waveforms with high crest factors.

Implicit computation

Implicit computation as shown in Fig. 8 may seem less direct than explicit computation, but it's simpler. There's no square-root needed, and this approach has better dynamic range than is the case in direct RMS computation. Here, A and B in the multiplier/divider are V_A, and C is V_{OUT}. Thus $V1 = (V_{IN})^2/V_{OUT}$. This voltage is filtered (averaged) to become the output, so

$$V_{OUT} = Avg[(V_{IN})^2/V_{OUT}],$$

or

$$V_{OUT} = \sqrt{Avg[(V_{IN})^2]}.$$

Analog Devices makes several IC's for implicit RMS-to-DC conversion, such as the general-purpose AD536A, the low-power/low-level AD636, the high-performance AD637, and the low-cost/low-power AD736/7, all used by several manufacturers of true-RMS DVM's. For an in-depth study of RMS-to-DC conversion, you can get a free copy of the *RMS-to-DC Conversion Application Guide* (see the box on page 57).

Figure 9 shows a practical circuit for a low-cost true-RMS meter using an AD536A as IC1. The input is scaled for 200 millivolts RMS at full scale, applied to a buffer amplifier on pin 7 (BUF IN) of IC1, and fed from pin 6 (BUF OUT) to an absolute value precision rectifier on pin 1 (V_{IN}). It then passes through a squarer/divider that provides a log output scaled in dB on pin 5.

The final output comes from a current mirror that produces a current of 40 μA DC per RMS volt of input. This current passes through an internal 25K resistor, to produce 1 volt DC/RMS volt of input. There's no offset or gain adjustment, since the AD2026 *D*igital *P*anel *M*eter (DPM) module includes them. If you use your own meter, you can add an output amplifier with gain and offset adjustments.

Current transformers

Electronic current measurement usually begins by passing a current through a dropping resistor to create a proportional voltage. For currents of tens, hundreds, or thousands of amps, this is impractical, requiring resistors in the milliohm range, or below. Current transformers reduce such high currents to practical levels, using a single winding of N turns on a toroidal core.

The wire carrying the current is threaded through the center of the toroid and becomes a single-turn primary. The secondary current is 1/N times the primary current, and is passed through a dropping resistor to create the proportional voltage. Current transformers generally produce 5 amps out at full-scale primary current.

Power and phase shifts

Up to this point, we've considered only true RMS measurement. Next, we'll examine the effect of phase shifts on power measurement. Our discussion will deal only with pure sinusoids. The concept of phase shift has meaning for sinusoids as well as other periodic waveforms of the general form:

$$V(t) = V_{PK} \times \sin[(2 \times \pi \times f \times t) + \theta].$$

Phase shifts are caused by capacitive or inductive reactances. In a pure capacitance, voltage lags current by 90 degrees, while in a pure inductance, current lags voltage 90 degrees. RC or RL circuits produce phase shifts other than 0 degrees or 90 degrees, while pure resistances cause no phase shift at all.

In Fig. 10-*a*, a charging current starts to flow into the capacitor as the voltage rises, stops when the voltage reaches its peak, and reverses as the voltage falls. If there were no resis-

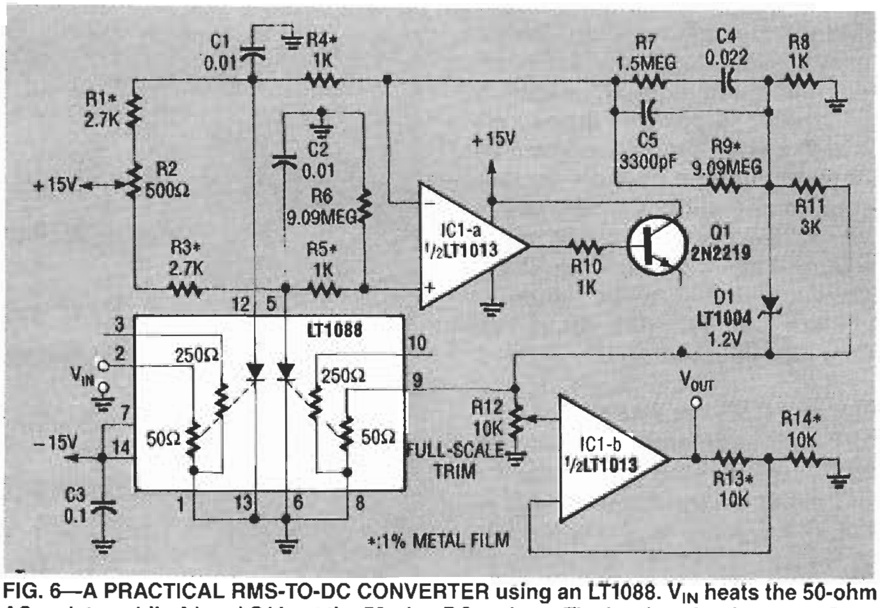


FIG. 6—A PRACTICAL RMS-TO-DC CONVERTER using an LT1088. V_{IN} heats the 50-ohm AC resistor while A1 and Q1 heat the 50-ohm DC resistor. The level at pin 9 is 1 volt DC per volt of RMS input, and A2 provides gain adjustment. Accuracy is 1% up to 50 MHz, 3-dB bandwidth is 300 MHz for A1, and input crest factors reach 50:1.

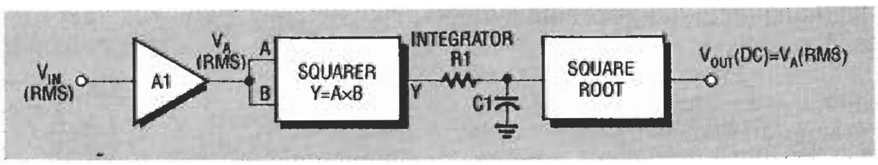


FIG. 7—DIRECT RMS computation; Amplified AC is rectified, squared, and integrated; taking the square root gives true RMS. The squaring multiplier gives full-scale output with both X and Y at full scale.

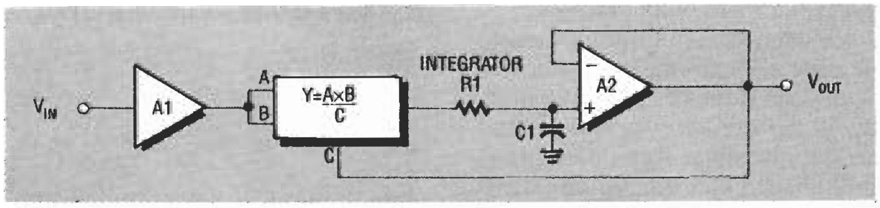


FIG. 8—IMPLICIT RMS COMPUTATION; it may seem less direct than Fig. 7, but it's simpler, as no square-root is needed, and has better dynamic range. Analog Devices makes several IC's for this role; the general-purpose AD536A, the low-power/low-level AD636, the high-performance AD637, and the low-cost/low-power AD736.

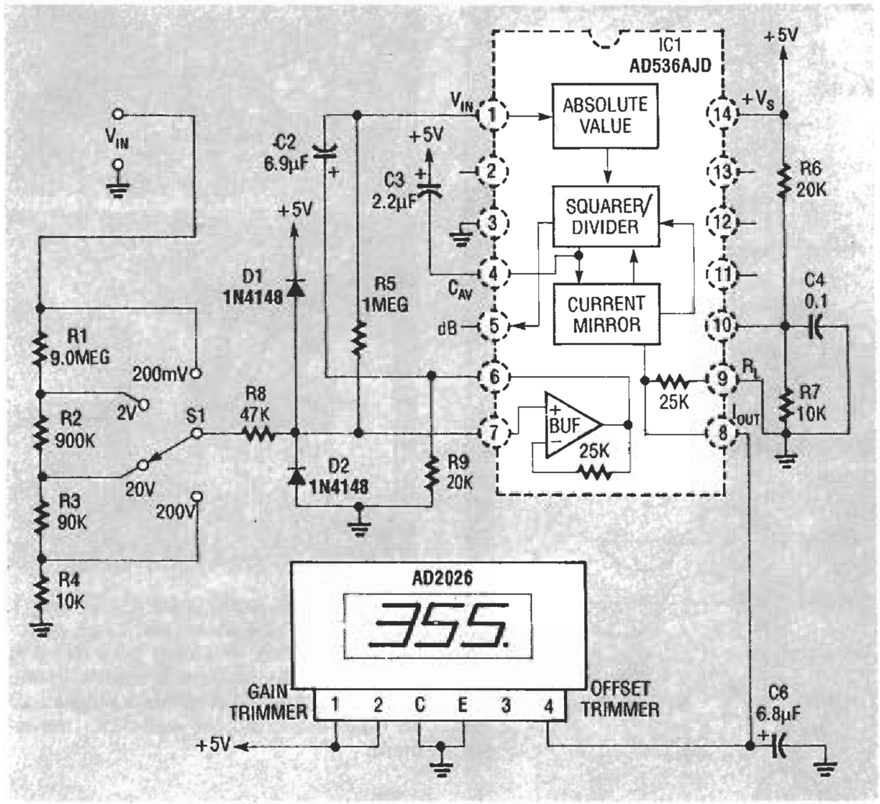


FIG. 9—A CIRCUIT FOR A LOW-COST TRUE-RMS meter using an AD536A. The current mirror sources 40 μA DC per RMS volt of input through an internal 25K resistor, producing 1 volt DC per RMS volt of input. There's no offset or gain adjustment, since the AD2026 *D*igital *P*anel *M*eter (DPM) module includes them.

tances and thus no power loss, the capacitor would store energy from the inductor part of each cycle, and feed it back the rest of each cycle, the generator acting like a motor. Looking at the waveforms, instantaneous power E × I is positive for half a cycle and negative the other half, so no net power is gained or lost.

Figure 10-*b* illustrates pure inductance. Any change in inductor current induces an EMF opposing the change, so current lags voltage by 90°. Energy is alternately stored in the magnetic field and fed back to the AC source. As in the case of the capacitor, no net power is gained or lost in an ideal inductive circuit with no resistance.

Resistances convert electrical current to heat; Fig. 10-*c* shows a pure resistance, and Fig. 10-*d* an RL circuit of under 90° phase shift. In general, power usage for purely sinusoidal circuits is P = E × I × cos(θ), where E and I are RMS voltage and current, θ is the relative phase, cos(θ) is the *P*ower *F*actor (PF), and E × I is measured in *V*olt-*A*mps (VA). Also, at θ = 0 degrees, PF = 1, while at θ = 90 degrees, PF = 0.

Ideal reactive loads don't consume power, but they increase total current drawn from a source, causing extra loss in the resistance of generators, transformers and power lines. Phase shifts are minimized by adding capacitance to balance inductive loads (motors and transformers). As mentioned earlier, the imaginary component of a joint resistive-reactive load corresponds to the reactive part. The fraction of the total power going into the reactive part of the load is measured in Volt-*A*mps *R*eactive (VAR), given by VAR = E × I × sin(θ). Also, at θ = 0 degrees, VAR = 0 VA, while at θ = 90 degrees, the VAR is maximized, with no power is being resistively dissipated as heat.

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Measuring phase shift

Phase shift provides a measure of the time difference between two waveforms, by detecting the time between zero crossings; Fig. 11 gives the general layout of a circuit that can be used to measure a time difference. Each comparator flips when its input crosses 0 volts; their polarities are opposite one another. Comparator B sets the flip-flop, while comparator A resets it, both on positive-going transitions.

If the voltage and current are in phase, the transitions will be ½-cycle apart, and the flip-flop will be set 50% of the time. Lagging current delays the set, producing longer low and shorter high intervals, while leading current does the opposite. The filter averages the flip-flop output, producing a voltage proportional to phase shift, and rises or falls indicating lead or lag.

This isn't a complete working circuit, as component values depend on input levels, supply voltages, and desired output. Gain and offset adjustments will be needed for calibration; the output stage is a good place to add them. One or both inputs should be transformer-coupled for safety, but remember that transformers add phase shift. Also, the output represents *theta*, not sin(θ).

Measuring true AC power

Now that we've covered true RMS measurement and phase shifts, we could consider a computational circuit for E × I × cos(θ), but that would be complex, and inaccurate for non-sinusoidal waveforms. It's easier to go back to the basic definition of power; at any given instant, power is P = E × I. Power is found by integrating (averaging) the E × I product over one or more cycles, as shown in Fig. 12-*a*.

Figure 12-*a* shows what takes place in an electromechanical wattmeter movement, while Fig. 12-*b* shows the

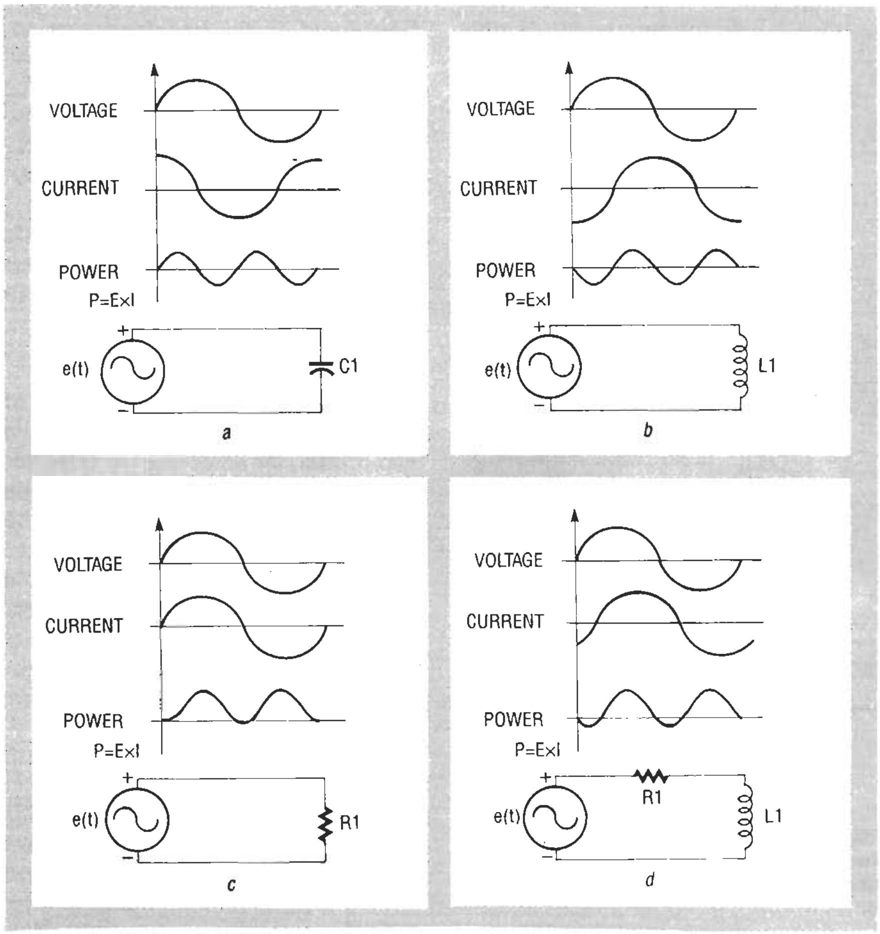

FIG. 10—A PRESENTATION OF VOLTAGE, CURRENT, and power waveforms, for capacitive, inductive, RC, and RL loads (*a–d*). With a 90° phase shift as in (*a*) and (*b*), average power consumption over a complete line cycle is zero. Voltage and current in phase, as in (*c*), produces maximum power. A 45° phase shift as in (*d*) reduces average power.

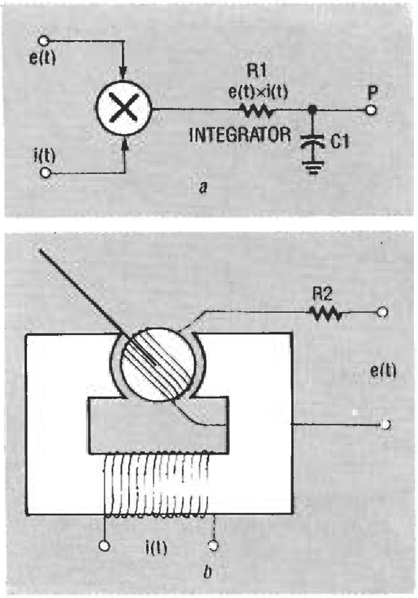

FIG. 12—TRUE POWER MEASUREMENT, UNAFFECTED by phase shifts and waveform distortions, is found by averaging e(t)×i(t). In (*a*) appears a sketch of the principle, while (*b*) displays a diagram of an electromechanical wattmeter movement.

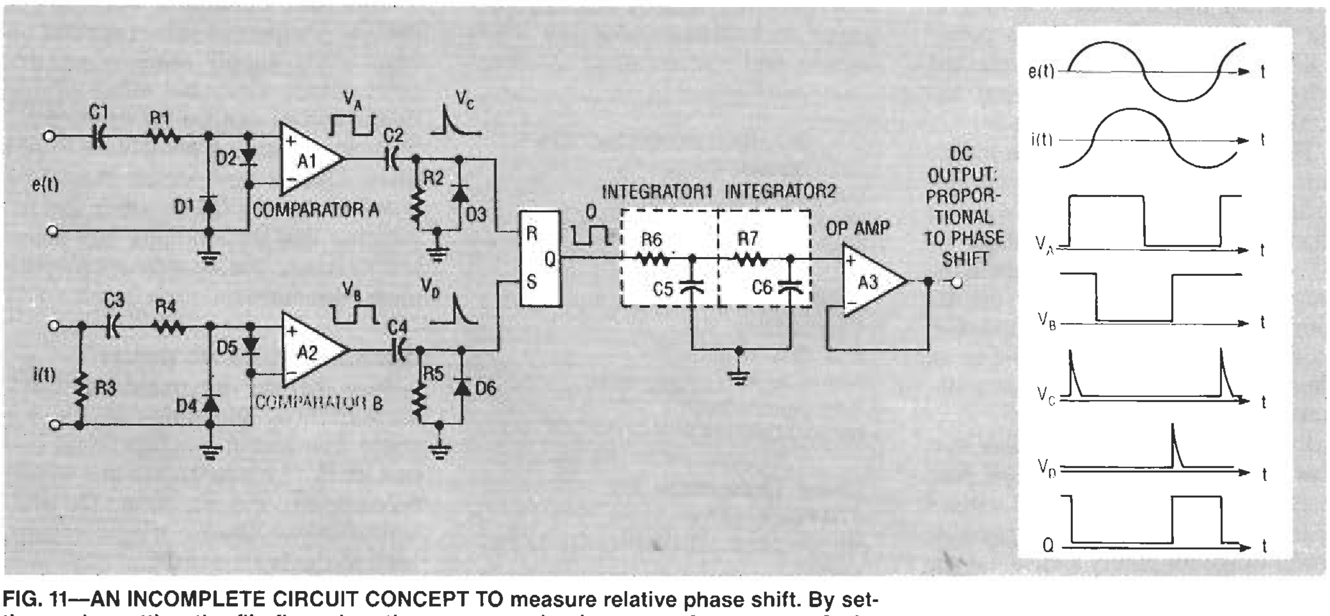

FIG. 11—AN INCOMPLETE CIRCUIT CONCEPT TO measure relative phase shift. By setting and resetting the flip-flop when the current and voltage waveforms cross 0, the relative phase shift can be measured.

movement itself. In the meter, one field is created by E, the other by I, and the attractive force between them is proportional to E×I. Filtering is achieved by the inertia of the meter movement plus mechanical damping; otherwise, the pointer would vibrate rapidly with each cycle.

Three types of electronic multipliers are used: log/antilog amplifiers, duty-cycle (pulse-width) modulators, and Hall-effect devices. Figure 13 shows the first, that performs multiplication by adding logarithms, log(E×I) = log(E) + log(I), and then takes the antilog, antilog[log(E×I)] = E×I. The final result is integrated to find average power.

The object here isn't to discuss log/antilog amplifiers; that's an article in itself. They all use the log relation

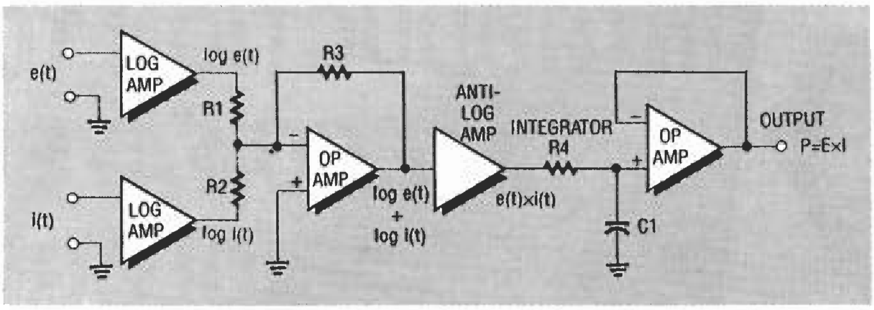

FIG. 13—MULTIPLICATION USING LOG/ANTILOG amplifiers. This circuit is also incomplete, but the log/antilog amplifier exploits the log relation between current through and voltage across the *B*ase-*E*mitter (BE) junction of a bipolar transistor to find average power.

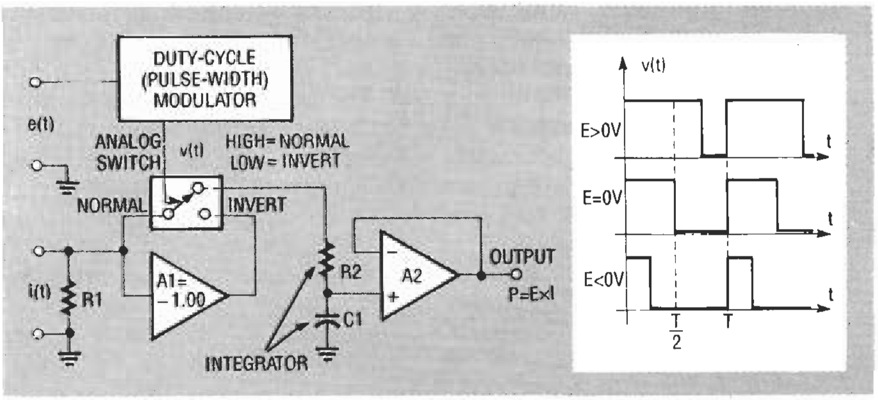

FIG. 14—AN INCOMPLETE DUTY-CYCLE (PULSE-WIDTH) modulator. The voltage input is converted to a duty cycle, modulating the current input to perform modulation.

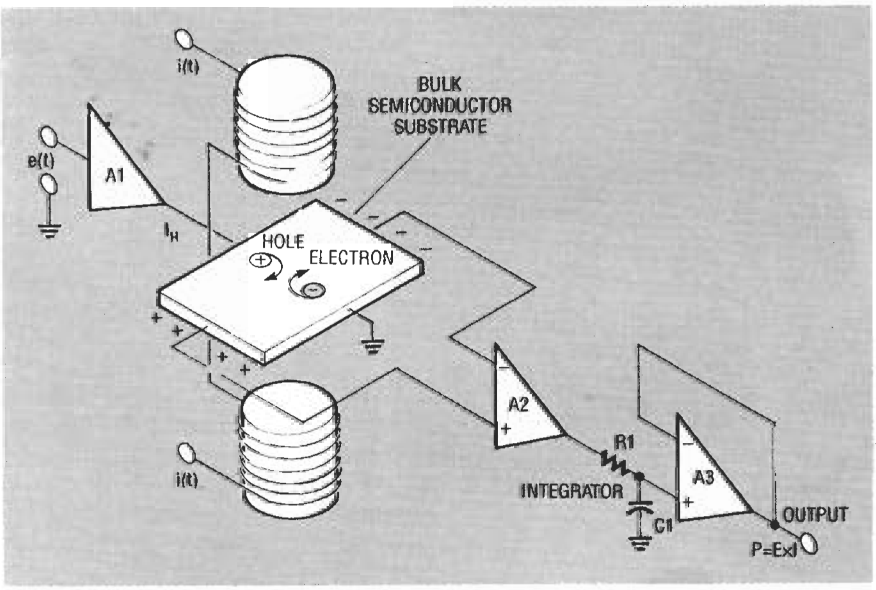

FIG. 15—A HALL-EFFECT DEVICE USED TO PERFORM multiplication, producing an EMF proportional to the product of the semiconductor's current times the magnetic flux density.

between voltage and current in the *B*ase-*E*mitter (BE) junction of a bipolar transistor. IC multipliers are available that perform the complete log-sum-antilog function internally. Examples include the Analog Devices AD530/630/830, Burr-Brown MPY100 and 4203, and Raytheon RC4200. Their prime advantages are simplicity, typical frequency response of 400 kHz or higher, accuracy of 0.25–1.0%, and temperature stability of 0.02–0.05%/deg C.

When extreme accuracy is needed at low frequencies such as 50–60 Hz, the duty-cycle (pulse-width) modulator shown in Fig. 14 gives superior performance. At 0 volts in, the pulse-width modulator produces a 50% duty cycle; its output controls an analog switch that alternately connects non-inverted or inverted inputs to the filter. At 50% duty cycle, the signal fed to the filter spends equal times positive and negative, so the filtered output is 0 volts regardless of the current input.

As the input goes positive, the duty cycle exceeds 50%. The switch spends more time in the normal position and less in the inverted, so the filtered output polarity is the same as that of the input current. Similarly, when the input voltage goes negative, the switch spends more time in the inverted position and the output polarity becomes opposite to that of the input current. The higher the voltage, the higher the duty cycle, and the higher the filtered output. The output is proportional to the product of duty cycle × I, or e(t) × i(t).

Unlike the log/antilog circuit, this one isn't available as an off-the-shelf IC, and is more complex to design and build. It's frequency response is also lower, since the duty cycle frequency must be many times higher than that of the inputs. Properly designed, however, it's capable of accuracies lower than 0.1%, limited more by the ability to obtain precise AC calibration sources than the circuitry itself.

Finally, we'll examine Hall effect multipliers, as shown in Fig. 15. A Hall-effect device uses a shaped and doped semiconductor in a magnetic field. As the holes or electrons flow at right angles to the field, they're deflected in a curved path toward the semiconductor edges, inducing an EMF between two edges proportional to the product of flux density B and current i(t); reversing either reverses the EMF.

In AC power measurement, the power current controls the magnetic field, and the power voltage controls the semiconductor current. The induced EMF is amplified by a differential amplifier, filtered, and scaled. Hall effect devices are very linear, and have a wide bandwidth. Their main drawback is they need temperature compensation to achieve high stability. Clearly, AC power measurement involves much more than multiplying voltage and current. The techniques shown here make possible precise measurement of RMS voltage/current, phase shift, and true power in electronic circuits. **R-E**