Convert coordinates and find SWRs graphically

by Vaughn D. Martin
Magnavox Co., Fort Wayne, Ind.

A relatively simple graphical procedure can reduce the error probability and the tedium of conventional mathematical approaches to finding standing-wave ratios, and converting admittance values in rectangular coordinates to impedance values in polar coordinates, including phase angles. The procedure requires only the known impedance, or admittance, and is executed with a pencil, a straightedge, and a compass. It involves just three steps and is more than 98% accurate.

For example, consider the circuit diagram (Fig. 1):

$R = Z_R = 50$ *ohms*
$G_R = 20$ *millimhos*
$C = 0.92$ *microfarad*
$X_C = 1/j(2\pi fC) = 1/j(5.78 \times 10^{-2})$ *ohms*
$B_C = 1/X_C = j57.8$ *millimhos*
$L = 1.2$ *millihenry*
$X_L = j(2\pi fL) = j75.4$ *ohms*
$B_L = 1/X_L = 1/j75.4 = -j13.3$ *millimhos*
$Y = G + jB = 20 + j57.8 - j13.3 = 20 + j44.5$

In graphical computations, admittance is represented by the hypotenuse of the right triangle in which conductance is represented by the base, and susceptance by the altitude. In many applications, however, admittance is more useful when expressed in polar coordinates. Graphical conversion is accomplished as follows:

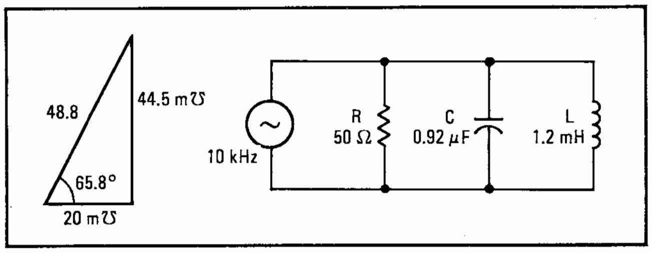
First, plot a point corresponding to the complex admittance on the chart (point A). Then, with a compass, draw an arc of a circle with center at the origin and passing through point A. The horizontal coordinate of point B, where the arc intersects the horizontal or conductance axis, is numerically equal to the total admittance; the impedance is indicated on the reciprocal scale by drawing a vertical line to that scale at point C, where the direct reading is 20.4 ohms. The phase angle is determined by the intersection of the graph's outer edge at point D with a line from the origin through point A. This value is about 66°. (Checking mathematically, the exact value is 65.85°.) The impedance, as determined from the chart, is

$Z = 20.4\angle 66°$

If the value of the susceptance is negative in the rectangular-coordinate form, the polar version is plotted in the same way, but the sign of the angle is negative.

Converting polar to rectangular coordinates is the reverse of this procedure. The first step is to draw a vertical line from point C, representing the impedance, 20.4, to point B on the X-axis. Then swing a 90° arc from point B to point E on the vertical axis, with the center point again at the origin. Finally, with a straightedge, draw a line from the origin to the known phase angle (point D) at the top of the graph. The admittance is read in rectangular form where the arc and this line intersect (point A).

The same chart can be used to determine the standing wave ratio (SWR) on a transmission line of known characteristic impedance Z_o for a mismatched load. The semicircles sweeping out from the 20-millimho point on the conductance axis are lines of constant SWR for a transmission line with Z_o = 50 ohms. If such a transmission line has a load of 100 ohms, it should have SWR = 2. The reciprocals of 50 and 100 ohms are 20 and 10 millimhos respectively. The 20 millimho point on the conductance axis represents the 50-ohm characteristic impedance; the load resistance's conductance of 10 millimhos, at point F, is one end of the semicircle for SWR = 2. The other end of the semicircle is at 40 millimhos (point G), corresponding to a load of 25 ohms.



For loads that are not purely resistive, the compass is used again. For example, if the point A is the load admittance, the arc through that point centered on the origin, just as in the coordinate conversion, cuts the conductance axis at B, which is about halfway between the semicircles for SWR = 2 and SWR = 3. This indicates a SWR of about 2.5, which agrees with the computed impedance of 20.4 ohms (50/20.4 ≈ 2.5).

Other sets of semicircles can be drawn for transmission lines of different characteristic impedances. In each set, the centers are on the conductance axis. The center of the smallest one is at the point corresponding to the characteristic impedance; each successively larger circle is centered at a coordinate which is half the sum of the two intercepts of that circle with the horizontal axis. These two intercepts, in turn, are the characteristic conductance multiplied and divided, respectively, by the SWR for that circle.

In the chart, for example, the circle for Z_o = 50 and SWR = 3 intercepts the horizontal axis at G•3 and G/3, or 60 and 6.7; its center is at ½(60 + 6.7) = 33.3. Likewise, for Z_o = 75 and SWR = 2.5, G = 13.3, the intercepts are at 33.3 and 5.32; the center is at 19.3. □

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