Gm REVISITED

Nothing to do with American car manufacturers, Gm is in fact a throwback from the days of tubes, now finding a new lease of life with up-to-date semiconductor devices. K.T.Wilson explains. . . .

MANY A LONG YEAR ago, when transistors were an item which hadn't been dreamt of by science fiction writers, we all used tubes, and we all knew the magic letters Gm. Gm stood for a quantity called mutual conductance, and it measured an important feature of the tubes from which we could work out how much voltage gain we could get out of a given bottle. Well, the years have passed, and tubes are dead for many purposes, but Gm lives and is back working for us.

It's odd that Gm should have gone out of fashion for so long, because the idea of Gm is even more useful in transistor amplifier circuits than it ever was in tube circuits. Still, the idea seems to be coming back in a big way, so let's take a look at it.

Mutual conductance of any electronic device means the ratio of signal current at the output to signal voltage at the input. For a transistor, this is the ratio $\tilde{I}c/\tilde{V}be$. $\tilde{I}c$ being the collector current and $\tilde{V}be$ the voltage between base and emitter, Fig 1. The squiggle above the letters means that it's AC signal voltage and currents we're talking about, not the steady bias voltages and currents,

Using Gm therefore allows us to represent a tube or transistor as a generator of signal currents, the amount of signal current being Gm $\tilde{V}in$. Now a current generator means a device which will deliver its current into any load, high or low. No tube or semiconductor is really like this, but for most of the uses we make of transistors, the idea of a current generator is not far from the mark.

Current Generators

If a transistor were a perfect current generator, it would have an infinite resistance at its output. That means just that a signal voltage applied between the collector and the emitter would cause no collector signal current.

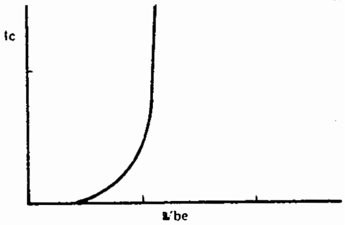


Fig. 1. Mutual conductance, Ic/Vbe for a transistor.

Once again, it's not quite correct but not far from the truth. A bit of collector signal current does flow, but not very much, about as much as would flow if there were a resistor of around 40k between collector and emitter.

Now the usefulness of all this is that it allows us to draw an equivalent circuit for a transistor. An equivalent circuit is a circuit made of simple components which behaves in just the same sort of way as some device which is, in reality, much more complicated. A simple equivalent circuit for a transistor is, therefore, as shown in Fig 2. It consists of a current generator, which generates a signal current Gm$\tilde{V}be$, and a resistor of about 40k in parallel. This simple circuit accounts for the size of the signal current at the output (the collector) and the output resistance between collector and emitter.

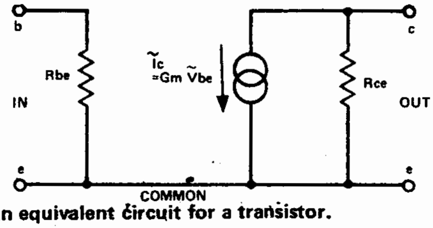


Fig. 2. An equivalent circuit for a transistor.

How does this help us? Quite a lot if we remember all the time that equivalent circuits are about signal currents, not about bias currents. As far as signal currents are concerned, the positive supply line of an amplifier is just as grounded as the ground line. Why? Because in the power supply there's a smoothing capacitor of several thousand microfarads, connected between the +ve and −ve lines. As far as DC is concerned, this capacitor is an insulator; but for AC signals the capacitor is just a short circuit, shorting the +ve line to the −ve line. When we connect a load resistor between the collector terminal of a transistor and the positive line, then, as far as *signals* are concerned the load resistor is connected between collector and emitter. Draw this into the equivalent circuit, and the result is Fig. 3. Back in the old days of tubes (nostalgia corner, this!), we found the sum of these two resistors in parallel, which was

$$\frac{Rce\ RL}{Rce+RL}$$

and then the voltage signal out was just the current signal times this resistance (Ohm's Law) giving

$$\frac{Gm\ Rce\ RL}{Rce+RL}$$

Simple Silicon

One of the things that makes life simpler in these days of silicon transistors is that the quantity Rce, the output resistance of the transistor, is quite a large value compared to most of the load resistors we use. An output resistance (the usual symbol nowadays is h_{oe}) of 40k is quite a bit larger than the 3k3 or so we use as a load, so that most of the signal current from the transistor is through this resistor in the equivalent circuit. That simplifies the output voltage to GmR_L so that the gain of a transistor amplifier is just GmR_L.

If it's as easy as that, why don't we see it in text books? The reasons are historical — we didn't start with silicon transistors, and a transistor, unlike a tube doesn't have a constant value of Gm. If we plot a graph of collector current against base voltage as in Fig. 1), the result is not the nice straight line we get when we plot such a graph for a tube or the not-too-crooked line we get when we plot the graph for an FET, but a very curved line indeed. This indicates that the value of Gm is not constant, but a value which changes as the current through the transistor changes. This, coupled with the rather low output resistance of the early germanium transistors seemed to seal the fate of Gm for good.

Ebers Moll

A few years back, though, the Ebers-Moll equation was noticed. You've never heard of it? You're not alone, very few text books mention it, and some mention it without explaining it. Very briefly, it's an equation which links the collector current with the Vbe value for a transistor. In other words, it's the equation for finding Gm. Now the full equation is a fearsome looking thing, full of mathematical symbols you may never have seen before. It repays close attention, though, because most of the symbols are of quantities that are pretty well constant, and only two of them vary very much. One of them is the steady bias current, Ic, and the other is temperature. As it happens, temperature, for the purposes of the Ebers-Moll equation, is measured in the Kelvin scale, which starts at the absolute zero of temperature around − 273°C. Room temperature is therefore around 293K (no degrees sign) in the Kelvin scale, and a few degrees above or below doesn't make much difference to the equation.

That leaves Ic as the one thing that really affects Gm, and the relationship works out at approximately

$$Gm=40Ic(Ic\text{ in mA})$$

Put in words, that means we can take a Gm value of 40 times the steady bias collector current in milliamps. For a bias current of 1 mA, the Gm value of a transistor is 40 mA/A. Too good to be true?Looks it, but it really does apply to any silicon transistor, apart from a few freak types.

This brings back the Gm idea in a big way, and we can forget a lot of the old formulae we once used in calculating the design of transistor amplifiers. The fact that Gm is not constant but varies with the bias current is, oddly enough, a help rather than a hindrance.

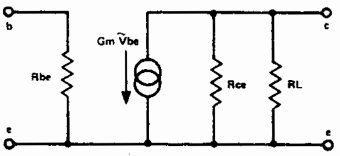


Fig. 3. For AC signals, a load resistor connected between collector and positive supply behaves as if connected between collector and emitter.

$$Gm=\frac{e}{kT}Ic$$

e = CHARGE CARRIED BY AN ELECTRON
k = BOLTZMANN'S CONSTANT
T = TEMPERATURE IN KELVIN SCALE
Ic = STEADY (BIAS) COLLECTER CURRENT

Gain

Going back to our equivalent circuit, and ignoring the large output resistance of the transistor, we can now write 40 I c in place of Gm (fig. 4). This makes the gain of a transistor with load resistor RL become 40 I c RL. But Ic in this equation is the *steady* bias collector current, and so IcRL must be the steady DC voltage across RL, the load resistor. This makes calculating the gain of transistor amplifiers with resistive loads a bit easier than falling off a log. Pick a value of voltage across the load resistor, multiply by 40, and that's your value of gain!

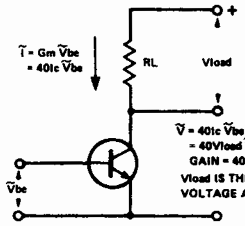
For example, we very often design voltage amplifiers so that about half of the supply voltage is dropped across the load resistor. For a 9 V supply, that's 4.5 V. Do this, and you can expect a voltage gain of 40×4.5=180 times. Don't believe it? It works all right, and tests on a single transistor amplifier confirm it as a rule of thumb. You don't, of course, expect to get a gain of *exactly* 180 in the case I've illustrated — there are 20% tolerances on load resistors apart from anything else, but you're never far out; that's what a rule of thumb is for.

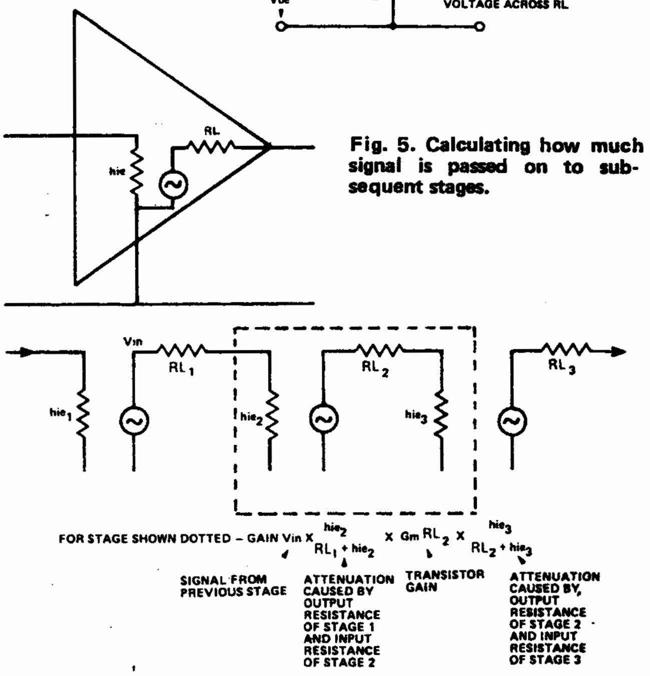
When you couple a single transistor amplifier to another stage, of course, that's another story. You may have set the gain of the first stage to 180 times, but not all of its ouput signal ends up usefully at the input of the next stage. Reason? The next stage has a rather low input resistance, and feeding signal from the collector of one transistor into the base of another, even if they are directly connected, is rather like feeding signal through a voltage divider. There are, in fact, two ways of calculating how much of the signal is passed on. One simple way is to imagine a voltage divider (Fig. 5) in which the load resistance of the first stage forms the upper resistor and the input resistance hie of the second stage. The quantity h_{ie} (in k ohms) is equal to h_{fe}/Gm, where h_{fe} is the current gain of the transistor, a quantity which *does* vary between one transistor and another. For a transistor with h_{fe}=100, Gm set to 40 (1 mA collector current) h_{ie} is 100/40=2k5. If we feed this from a transistor with a 4k7 load resistor, the amount of signal reaching the second transistor is

$$\frac{2.5}{2.5+4.7}=.35$$

of the signal at the output of the first. This brings the gain of the first transistor stage down to 180×.35=63 which is the sort of value we usually measure for one stage of a multi-stage amplifier.

Fig. 4. Transistor circuit with load resistor (RL). Gm can be replaced by 40Ic.





FOR STAGE SHOWN DOTTED – GAIN $Vin\times\frac{hie_2}{RL_1+hie_2}\times Gm\,RL_2\times\frac{hie_3}{RL_2+hie_3}$

Fig. 5. Calculating how much signal is passed on to subsequent stages.

With all this going for it, Gm, is coming back, folks. As Sam Goldwyn is supposed to have said, "simplicate and add lightness". Let's hope we've added a bit of lightness today.