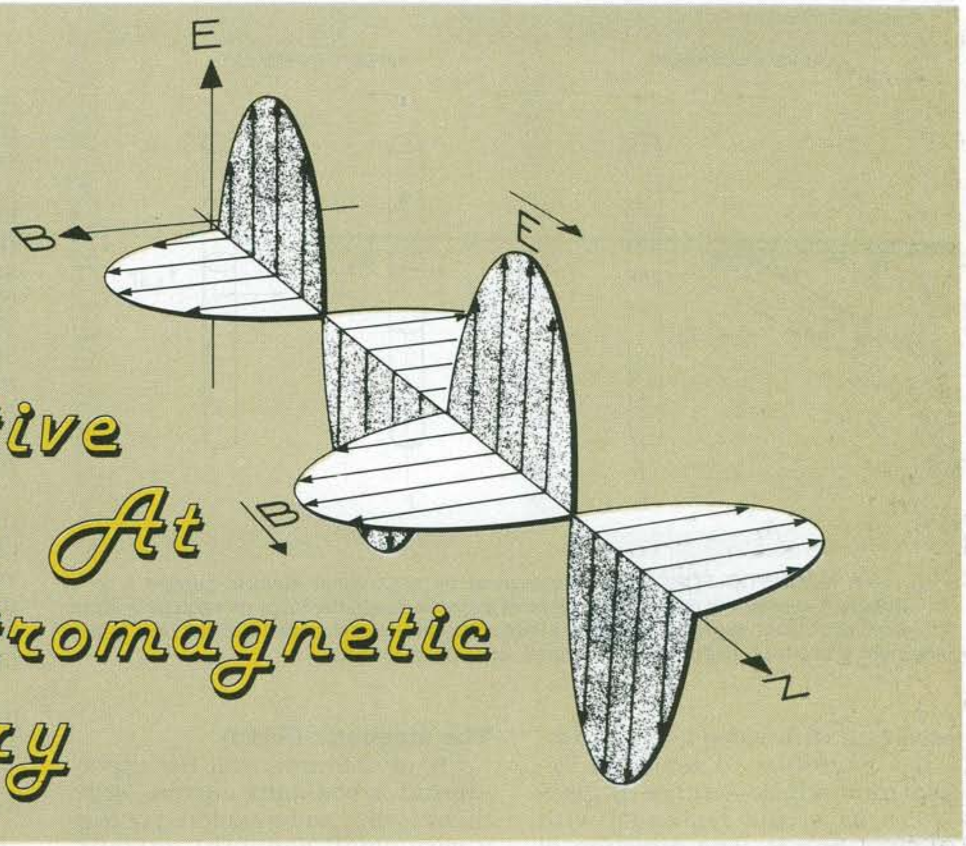
An Intuitive Look At Electromagnetic Theory



Learn the meaning behind magnetic charges and "B" fields.

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IN OUR LAST ARTICLE, WE DISCUSSED the general concepts of an electric field and how they applied to forces between static electric charges. We'll now develop an intuitive picture of how charges moving with a constant velocity produce an additional force, and how that force leads to the concept of a magnetic field.

Magnetic "charges"

Early experiments showed that if two permanent magnets were near each other, each experienced a force. In each magnet there appears to be two regions, called the north and south poles, that contain the source of the force. A pole of one magnet attracts the opposite pole of the other magnet but repels the other pole, thereby creating a torque. Apparently, a magnet produces something similar, but not identical, to that produced by an electric-charge distribution. A basic difference is that electric charge distribution can be separated into two distinct regions of positive and negative charge, while experiments show that cutting a magnet into smaller and smaller pieces simply result in more magnets, each having two poles. No matter how small a piece is taken, there's always an equal amount of north and south magnetic "charge." Experiments have shown that there's no such thing as an isolated single magnetic charge, or a magnetic monopole, only magnetic dipoles.

Hans Christian Oersted conducted experiments which showed that a permanent magnet near a conductor carrying a constant electric current I_1 experienced a similar force as shown in Fig. 1-*a*. Experiments by French physicist Andre Marie Ampere showed that when a conductor carries a constant current I along an infinitesimal length **dl** and another conductor caries a constant current I_1 along an infinitesimal length $\mathbf{dl}_1$, the length **dl** experiences an infinitesimal force in newtons

$$dF_m = k_m \frac{I\mathbf{dl} \times (I_1\, \mathbf{dl}_1 \times \mathbf{r}_1)}{r_1^2}$$

as shown in Fig. 1-*b*. $\mathbf{r}_1$ is a unit vector directed from $\mathbf{dl}_1$ to **dl** and r_1 is the separation distance. In the mks units, k_m is equal to

$$k_m = \mu_0/4\pi \text{ (webers/(ampere} \times \text{meter).}$$

$I\mathbf{dl}$ and $I_1\, \mathbf{dl}_1$, in units of m·C/s = A·m, are infinitesimal lengths of positive current in the direction of **dl** and $\mathbf{dl}_1$. In conductors, there are equal distributions of positive and negative electric charges even though the negative charges are moving. The **E** fields from the charges must sum to zero, so the $d\mathbf{F}_m$ must be distinct from the Coulomb force $\mathbf{F}_c$.

The infinitesimal force equation mentioned above is more complicated than for the static electric force since the direction of charge motion must be taken into account by vector multiplication, also known as the cross product ×. The direction of $I_1\, \mathbf{dl}_1 \times \mathbf{r}_1$ is defined by the right hand rule: curl the fingers of the right hand through the smallest angle from the vector $I_1\, \mathbf{dl}_1$ to the vector $\mathbf{r}_1$; the extended thumb points in the direction of $I_1\, \mathbf{dl}_1 \times \mathbf{r}_1$. The magnitude of $I_1\, \mathbf{dl}_1 \times \mathbf{r}_1$ is the area of a paral-

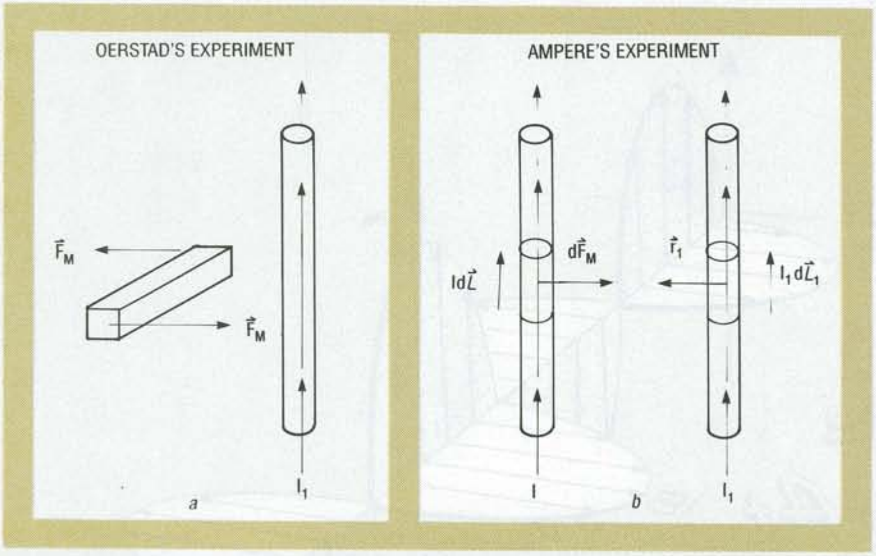

FIG. 1—A MAGNETIC FORCE F_m is produced by a constant electric current I_1 in a conductor. A permanent magnet experiences a torque due to the force on each pole (*a*). In (*b*) a small current segment I dl of another conductor experiences a force due to the segment I_1 dl_1. I_1 dl_1 experiences an equal and opposite force.

lelogram with sides I_1 $\mathbf{dl}_1$ and $\mathbf{r}_1$. That is similar to scalar multiplication where A times B gives the area of the rectangle with sides A and B. The direction of $\mathbf{dF}_m$ is that of the extended right hand thumb with the fingers wrapped through the smallest angle from I $\mathbf{dl}$ to I_1 $\mathbf{dl}_1 \times \dot{\mathbf{r}}_1$.

In Fig. 1-*b*, the current segment I $\mathbf{dl}$ experiences a force towards I_1 $\mathbf{dl}_1$. I_1 $\mathbf{dl}_1$ experiences an equal and opposite force towards I $\mathbf{dl}$. For other current segments, the force on an I $\mathbf{dl}$ is not equal and opposite to that on an I_1 $\mathbf{dl}_1$. That may appear to be a violation of Newton's third law, however the actual constant currents exist only in closed loops or circuits as dictated by charge conservation. The I $\mathbf{dl}$ and I_1 $\mathbf{dl}_1$ are only a part of each loop. The total force is found by summing up all the infinitesimal contributions around each closed loop. We must sum twice by integration, first to find the forces of all the I_1 $\mathbf{dl}_1$'s on an I $\mathbf{dl}$, and then to sum the forces on each I $\mathbf{dl}$

$$F_m = \frac{\mu_0}{4p} \oint \oint \frac{I Dl \times (I_1\ dl_1 \times r_1)}{r_1^2}.$$

The force on the entire loop composed of I $\mathbf{dl}$ is always equal and opposite to that on the entire loop composed of I_1 $\mathbf{dl}_1$.

Ampere went on to suggest that in a permanent magnet, the force $\mathbf{F}_m$ is produced by some sort of closed current loops that exist in the material.

The magnetic field B

Figure 2 shows that the space around a constant current segment I_1 $\mathbf{dl}_1$ can be explored using a very small constant current loop obtained by adding all its I $\mathbf{dl}$ contributions and symbolized by $\oint$ I $\mathbf{dl}$. Since each I $\mathbf{dl}$ will experience a force due to the presence of I_1 $\mathbf{dl}_1$ even though nothing material connects them, one has the impression that the condition of space itself is affected by the presence of the I_1 $\mathbf{dl}_1$. We can say that a constant current gives space the propensity to exert a force on another constant current, if it were present, according to Ampere's force law.

To find that propensity, we remove I $\mathbf{dl}$ from the force law to obtain the definition of the magnetic field (also called magnetic flux density) in units of webers/meter², which equals the tesla

$$dB = \frac{\mu_0}{4p} \frac{I_1\ dl_1 \times r_1}{r_1^2}.$$

This is called the Biot-Savart law. The force on each I $\mathbf{dl}$ is $\mathbf{dF}_m$, which equals I $\mathbf{dl} \times \mathbf{B}$. Opposite sides of the loop will experience forces in the opposite direction since the $\mathbf{dl}$'s are in opposite directions. The loop will, therefore, experience a torque. Since I_1 $\mathbf{dl}_1$ exists only as a part of a closed loop, the total $\mathbf{B}$ at any point in space is

$$B = \frac{\mu_0}{4p} \oint \frac{I_1\ dl_1 \times r_1}{r_1^2}.$$

Any current loop is called a magnetic dipole because it results in a $\mathbf{B}$ field.

The magnetic-field test instrument must be a very small magnetic dipole, just as a very small positive charge +q is the electric-field test instrument. The distinction is that $\oint$ I $\mathbf{dl}$ is a sum of all the vectors for which magnitudes and directions must be taken into account, whereas +q has only a magnitude.

If a current I is considered as just an individual electric charge q, moving with constant velocity through a point, the magnetic force it would experience in the $\mathbf{B}$ field at that point is

$$F_m = qv \times B.$$

If an electric field is also present, q would experience an additional electric force $\mathbf{F}_c$, and the total force would be

$$F = F_c + F_m = qE + qv \times B = q[E + (v \times B)].$$

This equation is known as the Lorentz force law.

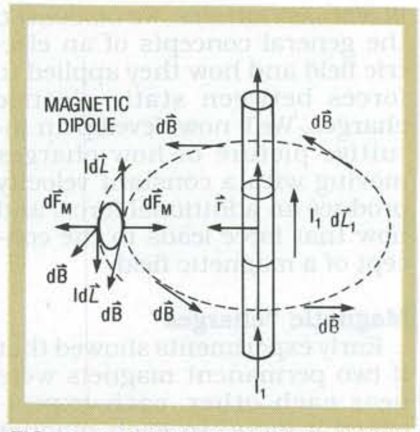

FIG. 2—A SMALL MAGNETIC DIPOLE, composed of current segments I dl, near a current-carrying conductor experiences a torque due to opposite forces dF_m on opposite sides. Since nothing material is pushing the dipole, the concept of the B field is used to account for the forces.

B field characteristics

The apparent flow of the $\mathbf{B}$ field from an infinitesimal volume about a point can be found by the same method used to find the electric flux. Imagine a Gaussian surface around a current loop composed of an infinite number of current segments I $\mathbf{dl}$ each producing a $\mathbf{dB}$ field as shown in Fig. 3-*a*. Divide the surface into an infinite number of infinitesimal **ds** areas. Through each **ds** there are an infinite number of **dB**'s. The total **B** field at each **ds** is $\mathbf{B} = \int d\mathbf{B}$ by linear superposition. Taking $\mathbf{B} \cdot d\mathbf{s}$ gives the magnitude of **B** times the magnitude of the effective area parallel to **B**. That is the apparent flow of **B** through **ds**. Summing those factors by integration over the entire surface gives the total apparent flow, or magnetic flux

$$\phi = \int \mathbf{B} \cdot d\mathbf{s} \text{ (webers).}$$

Imagine moving over the surface, adding up the $d\mathbf{B} \cdot d\mathbf{s}$ contributions from each I **dl**. At each **ds**, **r** points from I **dl** towards **ds**. Since **dB** is perpendicular to **r**, the only place $d\mathbf{B} \cdot d\mathbf{s}$ is non-zero is where **ds** is not directed along **r**. That is where **ds** moves away from or toward I **dl**. Since the surface is closed, for each place we move away from I **dl** by a certain amount and direction, there must be another place that we move back in towards I **dl** by the same amount and in opposite direction. Whatever $\mathbf{B} \cdot d\mathbf{s}$ contribution is found over some of the surface is canceled by a $-\mathbf{B} \cdot d\mathbf{s}$ contribution over another part of the surface, therefore we can say that

$$\phi = 0.$$

As in electric flux, any **B** produced by currents outside the surface will not contribute to the total.

If the original Gaussian surface is shrunk so the volume enclosed approaches zero, the ratio of the change in flux to the change in volume would reach a limiting value even if the flux were not zero. That is the divergence of **B**, and since $\phi = 0$ for any Gaussian surface

$$\nabla \cdot \mathbf{B} = 0 \text{ (T/m}^3\text{).}$$

That is the unnamed Maxwell equation. It simply says that the total spreading out, or divergence, of the **B** field through an infinitesimal closed surface about any point is zero. Whatever **B** field appears to leave from a particular point must return to that same point. Magnetic monopoles, therefore, cannot exist. That relationship allows magnetic dipoles, which produce equal amounts of outward and inward magnetic flux from a point. If a number of our **B**-field instruments were scattered about the point, they would not spread out.

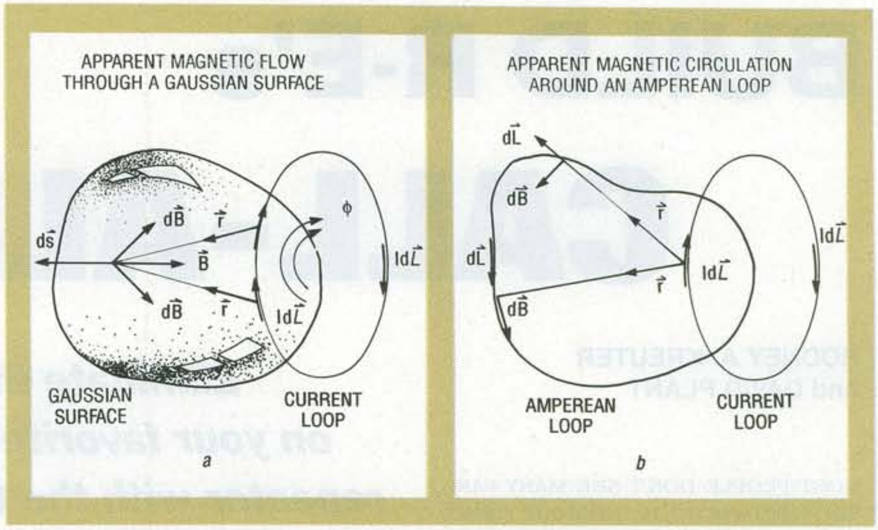

FIG. 3—CHARACTERISTICS OF A B FIELD. In (*a*), a Gaussian surface composed of an infinite number of infinitesimal ds areas surrounding part of a current loop. The total apparent flow of B, the magnetic flux ∫ B·ds, through the surface is zero. Flux from currents outside the surface does not contribute since whatever flux "flows" in through the surface also flows back out. In (*b*), an amperean loop composed of an infinite number of infinitesimal dL lengths encircles a current segment. The magnetic circulation around the loop ∮B·dL is proportional to the current encircled. For currents not encircled, ∮B·dL is zero.

The apparent rotation of the **B** field around an infinitesimal area containing a point can be found by imagining an amperean loop about some current loop as shown in Fig. 3-*b*. Divide the amperean loop into an infinite number of infinitesimal lengths **dL**. The **B** field at each **dL** is again just the sum of each of the infinitesimal **dB** contributions from each I **dl**, where $B = \int d\mathbf{B}$. The magnetic circulation around the loop is proportional to the current encircled.

If you take $\mathbf{B} \cdot d\mathbf{L}$ you get the magnitude of **B** times the magnitude of the effective length parallel to **B**, which is the apparent flow along **dL**. The direction of **dL** is taken as the direction of the curled fingers of the right hand with the extended thumb pointing in the direction of I **dl**. The total apparent rotation, also called the magnetic circulation, around the amperean loop is found by adding those parts by integration over the entire closed loop $\oint \mathbf{B} \cdot dL$.

Imagine moving along the loop, in the direction of **dL**, adding up the $d\mathbf{B} \cdot dL$'s. **r** points from I **dl** to **dL**. When we move at right angles to **dB**, that is along **r** or I **dl**, where $d\mathbf{B} \cdot d\mathbf{L}$ is zero. At all other places there will be a non-negative contribution since we are always traveling in one direction around the loop. The contributions are proportional to the current I through the loop since $\mathbf{B} = \int d\mathbf{B}$ is proportional to that current. The proportionality constant is μ_0, so

$$\oint \mathbf{B} \cdot d\mathbf{L} = \mu_0 I \text{ (T·m).}$$

If the amperean loop is shrunk so the area enclosed approaches zero, the ratio of the change in circulation to the change in area reaches a limiting value. That is the curl of **B**, which must be proportional to the current per unit area **J** through the loop, therefore

$$\nabla \times \mathbf{B} = \mu_0 \mathbf{J} \text{ (T/m}^3\text{).}$$

That relationship is called Ampere's law for constant currents. It simply says that the total apparent rotation, or curl of **B**, around any point is proportional to the constant current density at that point. The right hand rule gives the direction of apparent rotation. If a number of the **B**-field instruments were scattered about a point, they would rotate.

Next time, we'll discuss some magnetic phenomena and how inductance is related to the magnetic field. The concept of a magnetic circuit will be developed based on an analogy to the electric circuit. We'll see that in matter, the magnetic field can be considered as the linear superposition of two fields, similar to what was shown with the electric field. **R-E**