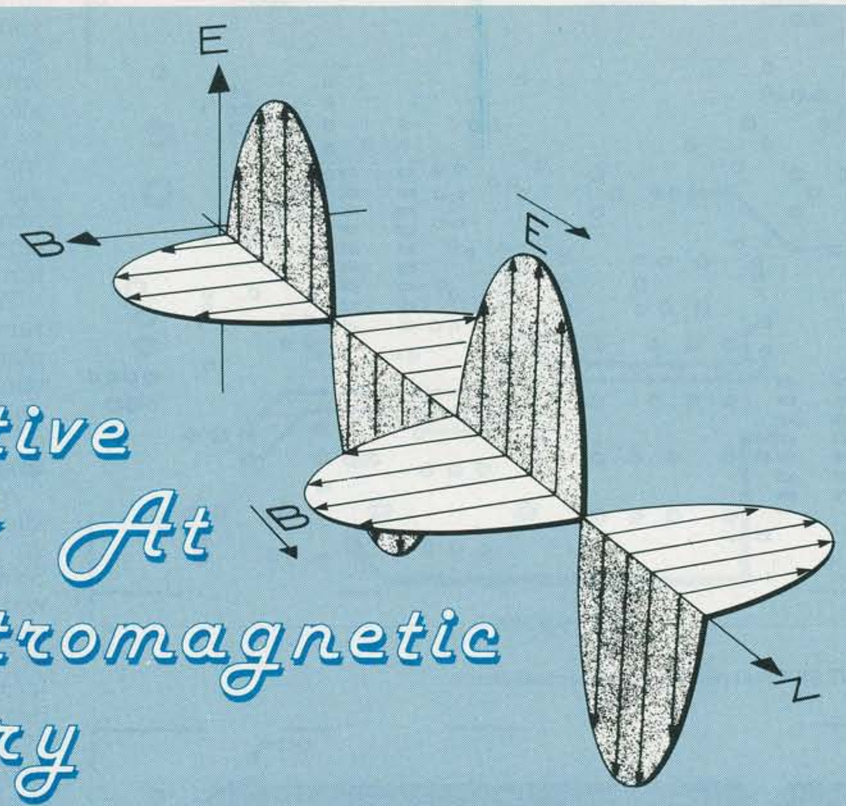

An Intuitive Look At Electromagnetic Theory

Find out more about magnetic phenomena and how inductance is related to the magnetic field.

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IN OUR LAST EDITION, WE DISCUSSED the characteristics of a static magnetic field in empty space. In this article we'll look further into the **B** field and its effects on matter. Of particular importance, we will show that the magnetic field in matter can be found by using the linear superposition of free and bound current densities.

Potential

If you recall, the expression $\nabla \times \mathbf{B} = \mu_0 \mathbf{J}$ says that the apparent rotation of the **B** field around a small region about a point is proportional to the current density in that region. Unless the current density or charge per unit area **J** is zero, **B** cannot be the gradient of a scalar potential and therefore is not a conservative field. However, in regions that have no current flow, $\nabla \times \mathbf{B} = 0$. In that case, the field is conservative and a scalar potential can be defined. Suppose a small current loop, the **B**-field instrument $\oint I\mathbf{dl}$, is moved quasistatically from point A to B in such a region as shown in Fig. 1. The force in the direction of motion $\mathbf{dL}$ gives the work done or change in magnetic potential energy

$$\Delta U_{ab} = -\int_a^b \left[(\oint I\mathbf{dl}) \times \mathbf{B} \right] \cdot \mathbf{dl} .$$

The work depends not only on the path taken but on the orientation of $\oint I\mathbf{dl}$ along the path. No work is done if $(\oint I\mathbf{dl}) \times \mathbf{B}$ is always perpendicular to $\mathbf{dL}$. Work is done if, at any place along the path, $\oint I\mathbf{dl}$ is rotated so that $(\oint I\mathbf{dl}) \times \mathbf{B}$ has some component parallel to $\mathbf{dL}$. That is the mechanical energy due to the work done against the torque.

Additional energy is required to maintain the current I in the loop. If the loop has resistance R, then I^2R is the rate of thermal energy loss. That energy must come from someplace, and if the magnetic field enclosed by the loop changes, more energy is required. We'll discuss the reason why additional energy is required in our next article.

Previously, we saw that any field with zero divergence is the curl of some other field. Since $\nabla \cdot \mathbf{B} = 0$, it must be that $\mathbf{B} = \nabla \times \mathbf{A}$. The **A** field is called the magnetic vector potential. It is not an energy field (energy is a scalar quantity), but it can be used in energy calculations. The main advantage in using the **A** field is that calculations required to solve many real-world problems are simplified. Since we won't be doing any calculations here, we will just say that the **A** field is real in the same sense as the **B** field.

We can use the analogy that the **A** field describes action at a distance from the **B** field just as the **B** field describes action at a distance from a current loop. The **E** field is also used to describe action at a distance from an electric charge. An appropriate instrument can be placed in a region of an **A** field, even through the **E** and **B** fields are

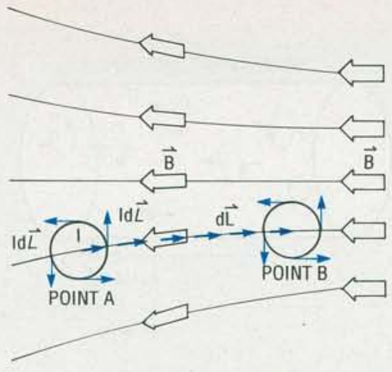

FIG. 1—A MAGNETIC DIPOLE IN A B FIELD is moved from point A to B along the path composed of d_L. The force vector on any small segment of the current loop is $dF_m = Idl \times B$. dF_m is directed out of the page as is the total force $F = \oint Idl \times B$. The force vector is perpendicular to dL, so the work done on $F \cdot d_L$ is zero. If the dipole is rotated so that F was not normal to the paper, then work would be done.

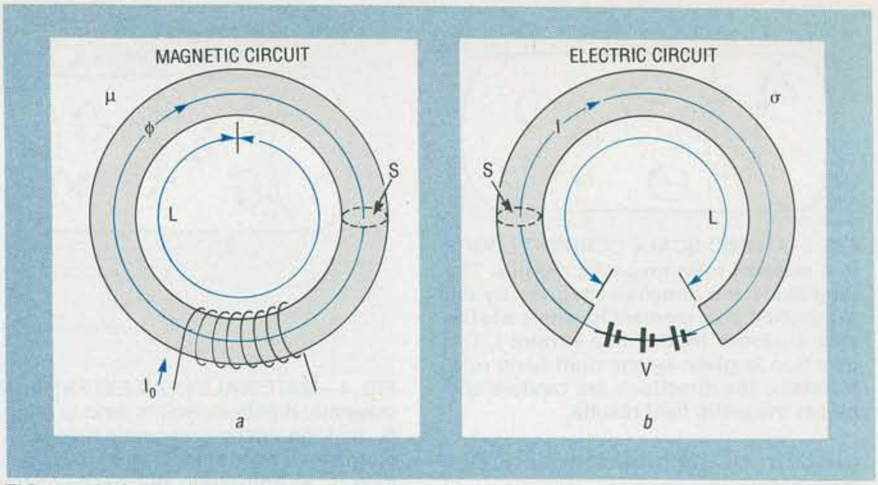

FIG. 2—MAGNETIC FLUX IS ANALOGOUS TO ELECTRIC CURRENT. In (*a*) the magnetic path of length L and cross-sectional area S is in a material of permeability μ. The source of magnetomotive force nI_o is the current I_o encircling the material n times. In (*b*) the electrical path is in a material of conductivity σ. The source of electromotive force nV is a battery of n cells each with a voltage V.

zero there, and an influence can be measured. The Bohm-Aharanov effect is an example.

Magnetic "current"

Recall that $\nabla \cdot \mathbf{B} = 0$ says that the lines of magnetic flux are closed lines. Nothing material flows along these lines but we can make an analogy with the closed path of a constant electric current. The magnitude of **B** in the magnetic circuit of Fig. 2–*a* can be found from $\oint \mathbf{B} \cdot dL = \mu I$, where L is the total length of the magnetic path, μ describes a property of the path material to be discussed later, and I is the total electric current enclosing the path. There are n turns of wire each carrying current I_0 so $I = nI_0$. Since the material is uniform, the magnitude of **B** must be independent when d**L** is being summed. So, denoting the magnitude of **B** as B and summing by integration gives

$$BL = \mu n I_0.$$

The magnetic flux is

$$\phi = \int \mathbf{B} \cdot d\mathbf{s}$$

where **s** is the cross-sectional area of the path. Since the area is uniform

$$\phi = BS = \frac{nI_o}{L/\mu S}.$$

In the circuit shown in Fig. 2–*b*, a current I exists in a material of length L, conductivity σ, and cross-sectional area S. The voltage is supplied by n cells, consisting of V volts each. From Ohm's law

$$I = \frac{nV}{L/\sigma S} = \frac{nV}{R}.$$

The so-called magnetomotive force nI_0 can be compared to the voltage nV. The magnetomotive force is summed in the same way voltages are summed. μ is similar to σ, which suggests that $L/\mu S$ is a magnetic resistance R_M, called reluctance. Those facts, along with the motivating fact that electric current and magnetic flux form closed paths (implying a conservation of something), allow analogous magnetic circuit equations to be developed.

Magnetic field in materials

In any material there are small current loops or magnetic dipoles formed by the atomic-scale rotational and orbital motions of the electrons and charges in the nuclei, as shown in Fig. 3. The vector quantity I**s** (where **s** is the area of each atomic-current loop), is the magnetic dipole moment. Normally the magnetic dipole moments have random orientations, so no average or macroscopic magnetic field is present.

When a material is placed in an external magnetic field $\mathbf{B}_o$, the quantum-wave functions are changed in such a way that there is a higher probability of the magnetic dipole moments being aligned antiparallel to the $\mathbf{B}_o$, as shown in Fig. 4–*a*. The directions may not all exactly align and may not be uniform except in what we call simple magnetic materials. The net effect is that magnetic poles appear at the ends of the material. We say the material has an induced magnetic field, a magnetic polarization, or simply that it is magnetized. This induced magnetic field is called the demagnetization field $\mathbf{B}_d$. The total magnetic field in the material is $\mathbf{B}_i = \mathbf{B}_o + \mathbf{B}_d$. $\mathbf{B}_d$ is antiparallel to $\mathbf{B}_o$ so $\mathbf{B}_i$ has a smaller magnitude than $\mathbf{B}_o$. Such a material exhibiting those characteristics is called diamagnetic.

In some materials there are additional magnetic dipoles resulting from electrons with unpaired spins. Their magnetic dipole moments are normally oriented randomly. When placed in an external magnetic field, the wave functions are changed in such a way that there is a higher probability of the magnetic dipole moments being aligned parallel to the $\mathbf{B}_o$ as shown in Fig.–4*b*. $\mathbf{B}_d$ is aligned parallel to $\mathbf{B}_o$, so $\mathbf{B}_i$ has greater magnitude than $\mathbf{B}_o$. A material exhibiting those characteristics is called paramagnetic.

In many materials, when the external $\mathbf{B}_o$ field is removed, the wave functions return to their original form within a short time and $\mathbf{B}_d$ becomes zero. However, in ferromagnetic materials the wave functions don't return completely and in some regions, called magnetic domains, residual alignment remains. It is as if each domain supplies a $\mathbf{B}_o$ to all other domains, thus maintaining some $\mathbf{B}_i$ in each.

$\mathbf{B}_d$ is not a particularly useful quantity. If there are n magnetic dipoles per unit volume, then a

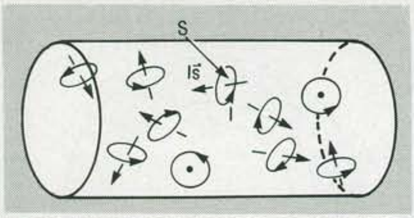

FIG. 3–ATOMIC-SCALE CURRENT LOOPS in a material form magnetic dipoles. The magnitude and direction are given by the magnetic dipole moment Is, where s is the area enclosed by the loop current I. The direction is given by the right-hand rule. Normally, the directions are random and no net magnetic field results.

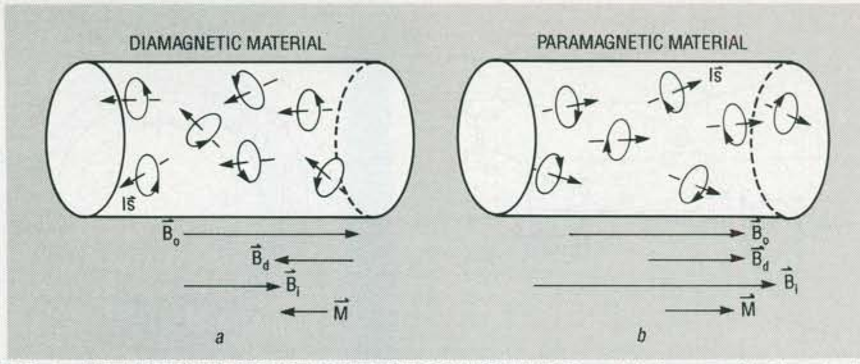

FIG. 4—MATERIALS IN AN EXTERNAL MAGNETIC FIELD B_o exhibit magnetization. In (*a*), magnetic dipole moments tend to align antiparallel to B_o. Demagnetization B_d opposes B_o and the internal magnetic field B_i is smaller in magnitude than B_o. In (*b*), the dipole moments tend to align parallel to B_o due to unpaired electrons. B_i is greater in magnitude than B_o. In both cases the magnetization per unit volume M is related to B_d. The vectors are shown outside of the material for clarity.

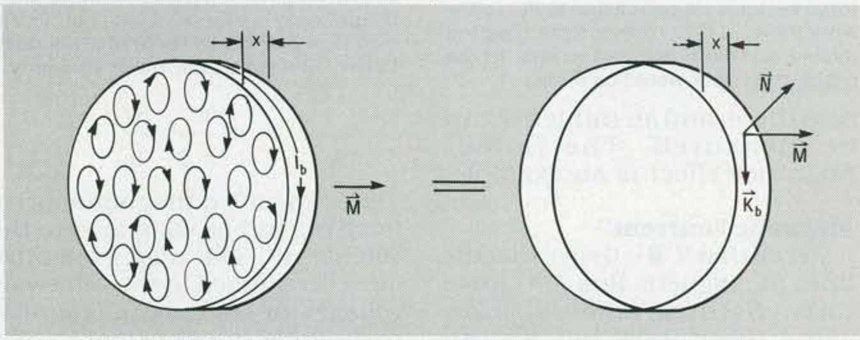

FIG. 5—ELECTRIC CURRENTS associated with individual magnetic dipoles cancel inside the material. At the surface, however, the currents are in the same direction resulting in a net surface current I_b. I_b is bound to the surface since it consists of pieces of the dipole currents bound in the material.

measure of the total magnetic polarization is

$$\mathbf{M} = n(I\mathbf{s})\zeta \text{ (A/m)}$$

called the magnetic dipole moment per unit volume (or just magnetization). ζ is a function of the average alignment of the dipoles with the external field and takes on values from −1 for total antiparallel alignment to +1 for total parallel alignment. $\mathbf{B}_d$ and **M** are related by a factor that takes into account properties of the material.

We can use the idea of Ampere's law, which says the apparent rotation of a magnetic field around a small region is proportional to the current per unit area in that region, to account for the **M** field. On an average, the atomic-scale magnetic-dipole currents cancel everywhere in a material except at the surface, as shown in Fig. 5. **M** can therefore be attributed to a bound surface current I_b around an area of magnitude S in a material of length x. The magnitude of **M** is simply the magnetic dipole moment per unit volume as illustrated by

$$I_bS/(xS) = I_b/x.$$

It's sometimes convenient to define a lineal-surface current density as

$$K_b = \mathbf{M} \times \mathbf{N} \text{ (A/m)}$$

where **N** is a unit vector normal to the surface. The curl of **M** is found the same way Ampere's law for static currents was derived, except the current density of concern is the average atomic-scale volume current density bound in the material $\mathbf{J}_b$. That gives us the formula:

$$\nabla \times \mathbf{M} = \mathbf{J}_b \text{ (A/m}^3\text{)}.$$

A convenient way to separate the external and internal contributions is to consider the total current density **J** as a linear superposition of $\mathbf{J}_b$ due to the material and all other currents called the free current density $\mathbf{J}_f$. From Ampere's law, it can then be concluded that

$$\mathbf{J}_f = \mathbf{J} - \mathbf{J}_b = \frac{1}{\mu_o}(\nabla \times \mathbf{B}) - \nabla \times \mathbf{M} = \nabla \times \left[\frac{1}{\mu_o}\mathbf{B} - \mathbf{M}\right].$$

The term in brackets is called the magnetic-field intensity or just the magnetic field (not to be confused with the **B** field)

$$\mathbf{H} = \frac{1}{\mu_o}\mathbf{B} - \mathbf{M}.$$

In simple materials, **B** and **M** are along the same line so $\mathbf{B} = \mu_o(1 + \varkappa_m)$ and $\mathbf{H} = \mu\mathbf{H}$. $\varkappa_m$ is called the magnetic susceptibility and μ is the magnetic permeability of the material. A commonly used quantity is the relative permeability which can be written as

$$\mu_r = 1 + \varkappa_m = \mu/\mu_o.$$

μ_r is less than 1 for diamagnetic materials and greater than 1 for paramagnetic materials. In ferromagnetic materials, μ_r is very large but the **H** and **M** relationship is generally more complicated and μ_r is not a simple constant.

Ampere's law now says

$$\nabla \times \vec{H} = \vec{J}_f$$

This says that the apparent rotation of the H field around a small region is due to the density of free current through that region. One of Maxwell's great contribuiton was the modification of Ampere's law.

Inductance

We know that a conductive loop, enclosing empty space or some material, forms an inductor. If the loop is carrying a constant current I, then a proportional magnetic flux exists through the area **s** enclosed by the loop. The constant of proportionality is the inductance, in units of webers per ampere, or henrys

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$$L = \Phi/I = (\int \mathbf{B} \cdot d\mathbf{s})/I \text{ (H)}.$$

Since **B** may not be constant across the area, we sum each infinitesimal contribution by integration. Note that we're concerned with the flux through the enclosed area, not the total flux through a Gaussian surface enclosing the loop, which is zero. For simple materials, L is independent of I since the equation $\mathbf{B} = \mu\mathbf{H}$ is proportional to I. However, L is dependent upon the area since the equation $\int \mathbf{B} \cdot d\mathbf{s}$ depends on the total area being summed. The inductance (L) is also dependent on μ.

We can use Ampere's law to see that effect. In empty space, $\mathbf{M} = 0$ and there are no bound currents, so we can say

$$\nabla \times \mathbf{H} = \nabla \times \mathbf{B}/\mu_0 = \mathbf{J}_f$$

and

$$\nabla \cdot \mathbf{H} = \nabla \cdot \mathbf{B} = 0.$$

With a simple material filling space, **H** = **B**/mu, so

$$\nabla \times \mathbf{H} = \nabla \times \mathbf{B}/\mu = \mathbf{J}_f$$

and

$$\nabla \cdot \mathbf{H} = \nabla \cdot \mathbf{B}/\mu = 0.$$

Since the divergence and curl of the field completely characterize the fields, **B** is larger by $\mu/\mu_0 = \mu_r$ in a filled inductor.

In our next article, we'll look at the effects of electric and magnetic fields as they change with time. We'll see that these fields are so closely related to each other that they lead to a single electromagnetic field. **R-E**